# COMP 414/514: Optimization – Algorithms, Complexity and Approximations

Lecture 11

# Overview

$$\min_{x} \quad f(x)$$

$$\text{s.t.} \quad x \in \mathcal{C}$$

- Different objective classes
- Different strategies within each problem
- Different approaches based on computational capabilities
- Different approaches based on constraints

And, always having in mind applications in machine learning, AI and signal processing

The focus of this lecture

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Unconstrained optimization

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- In this lecture, we will:
  - Discuss how to **distribute optimization in large-scale settings**
  - Study **synchrony vs. asynchrony** in gradient descent
  - Provide some rough theoretical results on how asynchrony affects performance
  - Alternatives and state of the art

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  $iii)$ The whole process is sequential

$$x_{t+1} = x_t - \eta \nabla f_{i_t}(x_t) = x_{t-1} - \eta \left( \nabla f_{i_t}(x_t) + \nabla f_{i_{t-1}}(x_{t-1}) \right) = \cdots = x_0 - \eta \sum_j \nabla f_{i_j}(x_j)$$

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- How can we distribute this computation over multiple processing units?

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- **Multi-node distributed computing**:
    - *i*) Many machines (up to 1000s), probably with many cores each
      *ii*) Shared-nothing architecture (each machine has its own CPU, storage)
      *iii*) Communication between nodes is much less cheap than single node

# Distributing gradient computations

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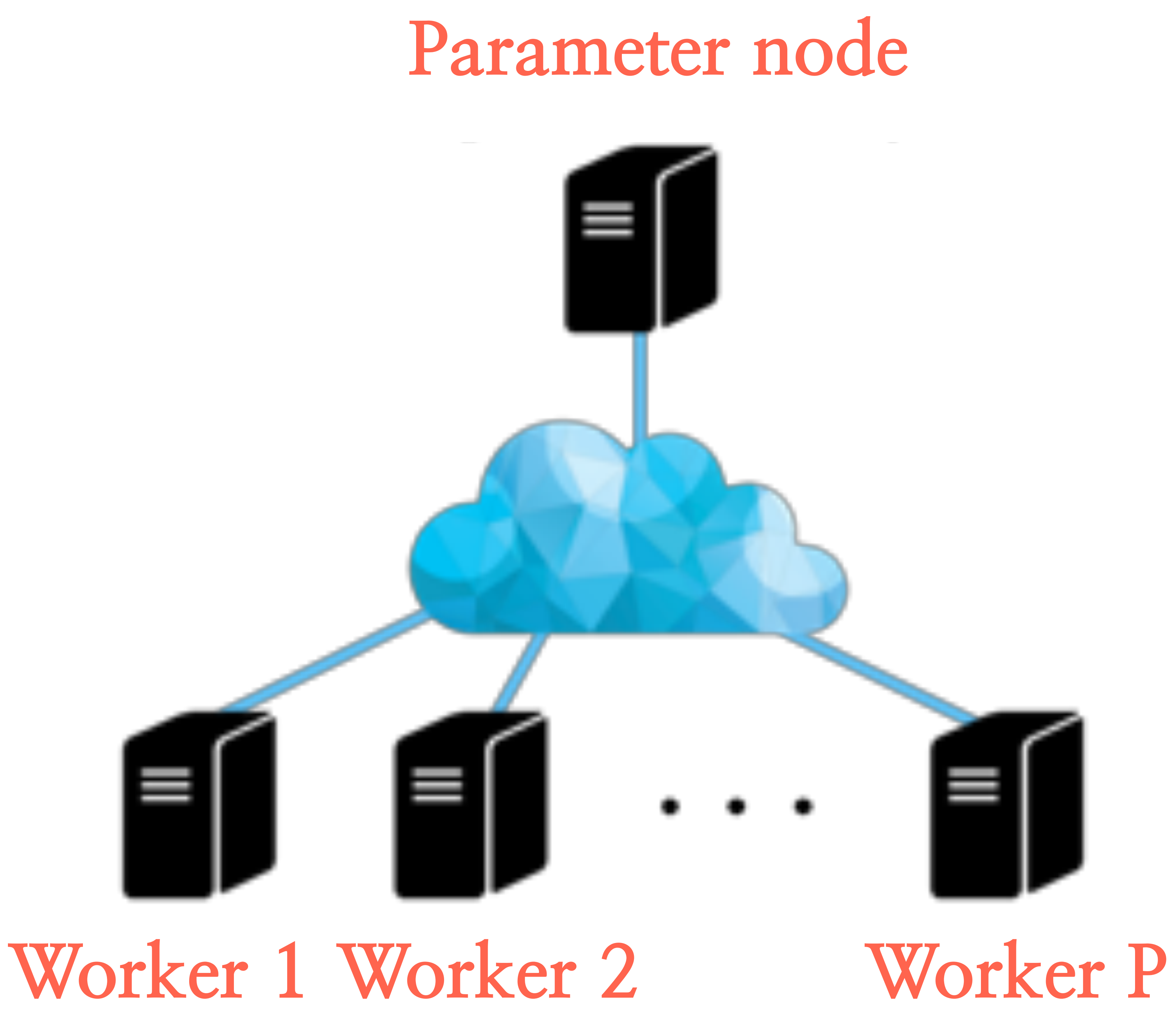


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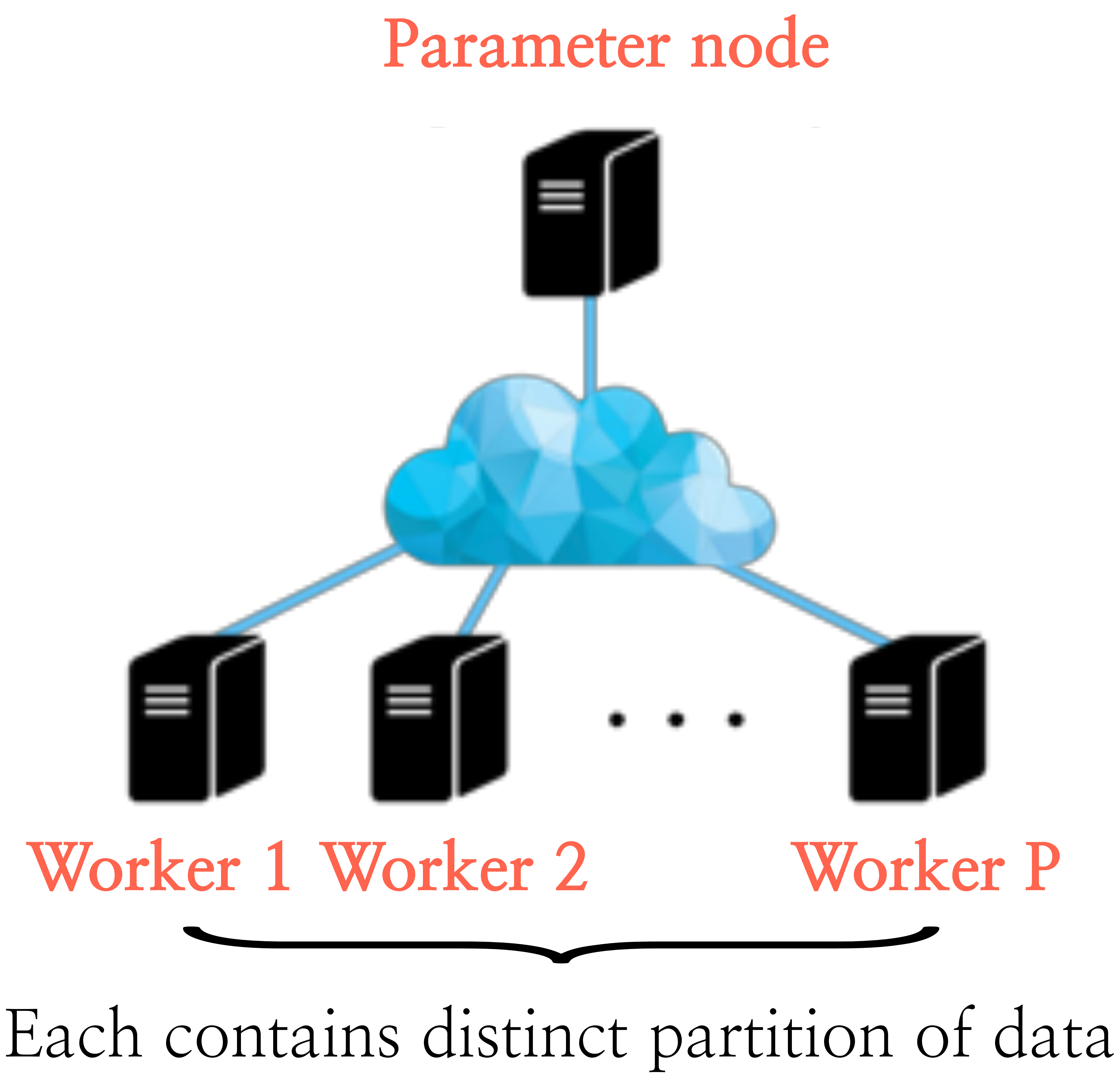


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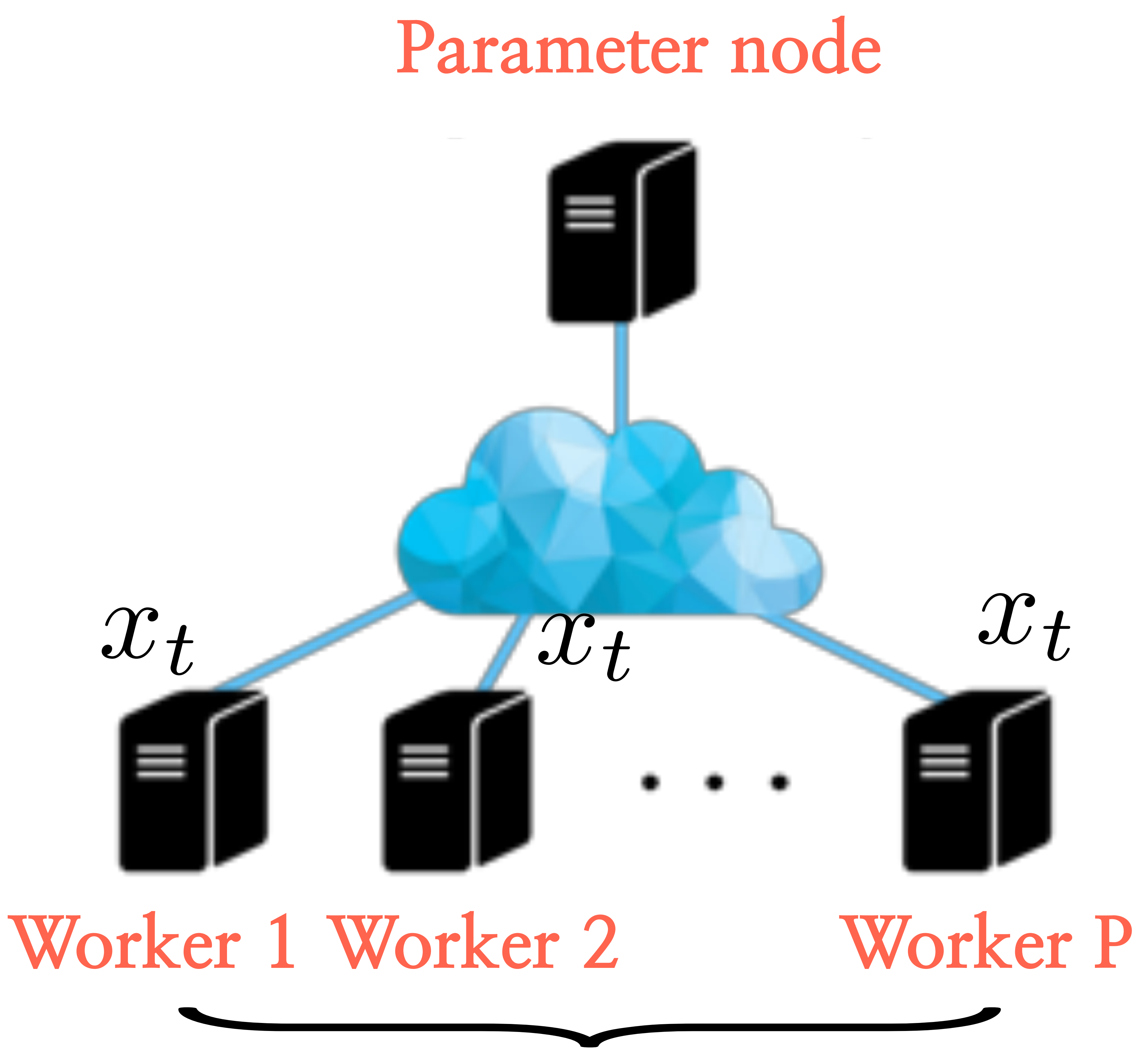


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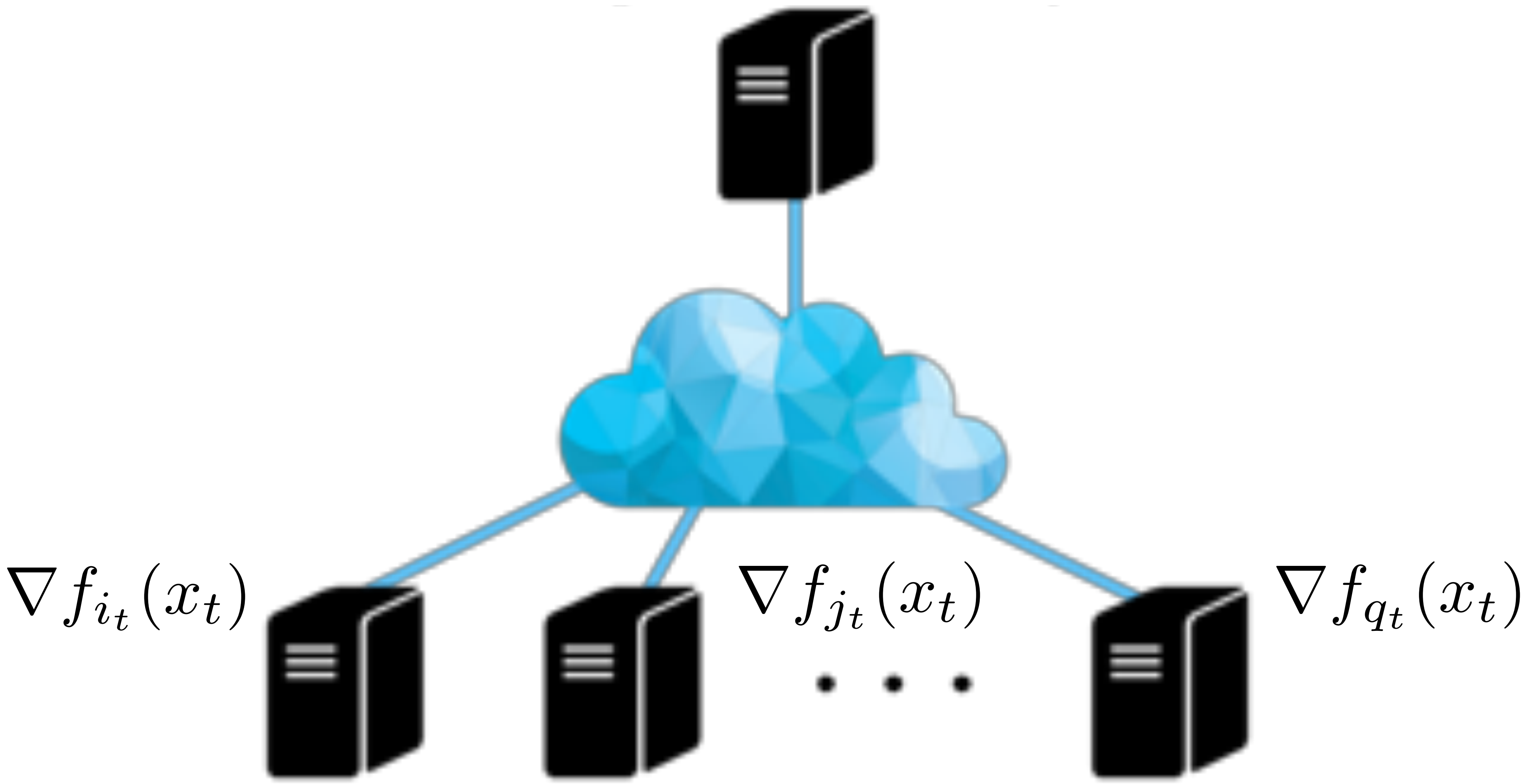


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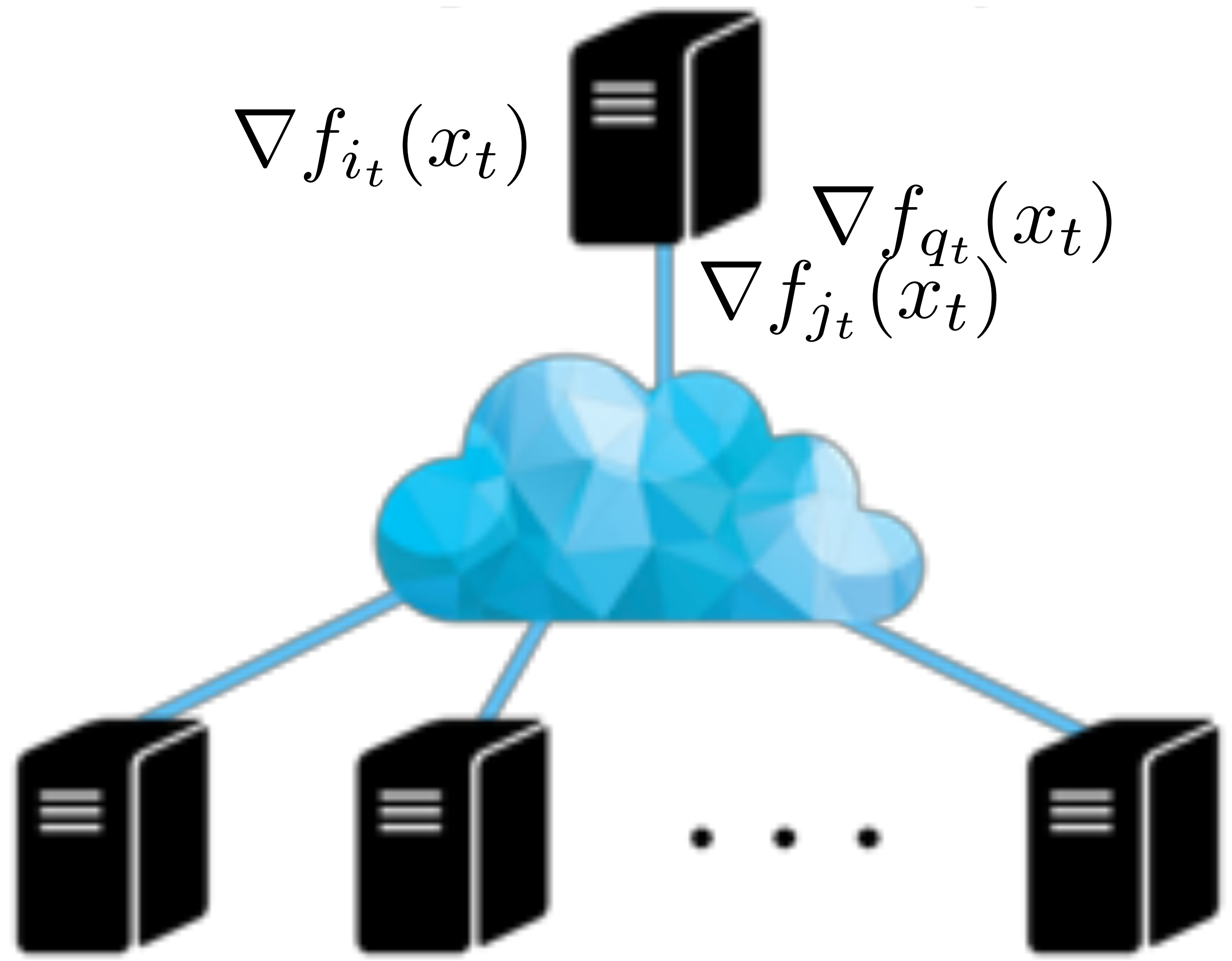


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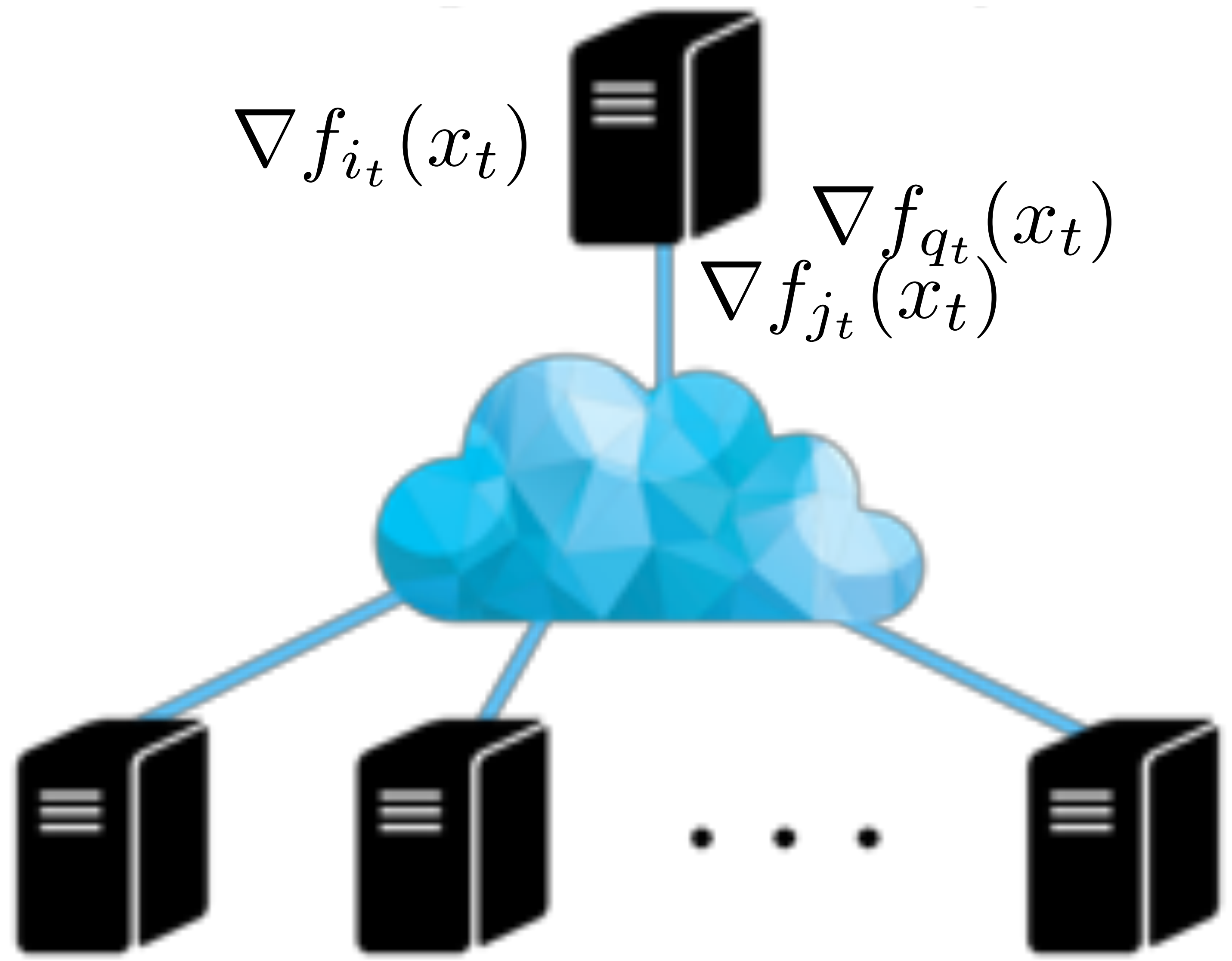


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(..till the very last slow worker – active research: tackle stranglers)

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- ``Well, it might be the case that we don't have all data at once``

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- ``But, there are cases where we have finite & fixed data – see neural networks``
- ``Well, the problem here is that full gradient descent does not perform well``

**Generalization vs. training error** :

1. If we care about only the training error, full GD could work well
2. In ML tasks, we often care about the generalization error, i.e., the performance of the model on unseen data

# Training vs. generalization error and GD

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Whiteboard

(This relates to the question "large vs. small batch training")

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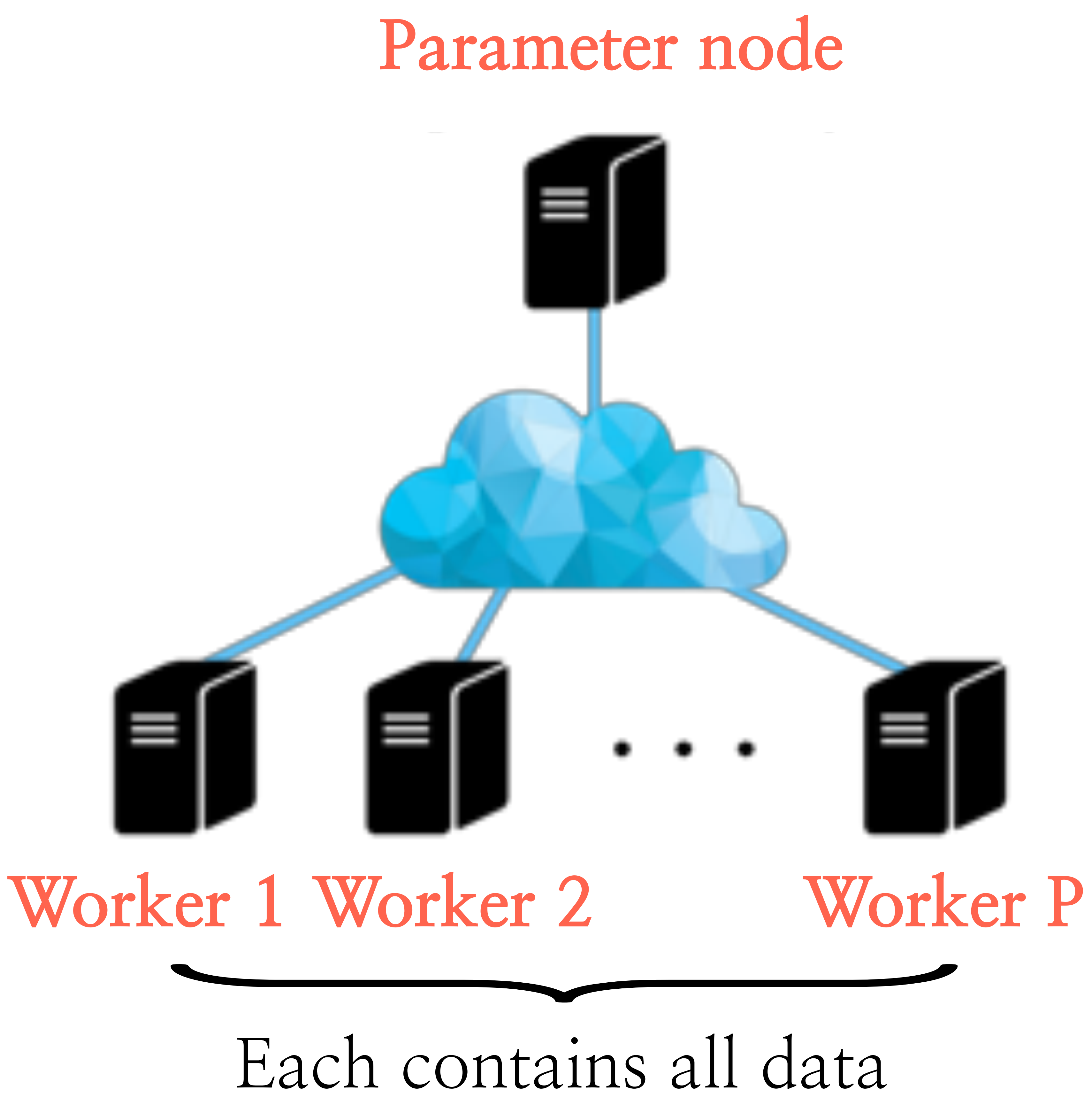

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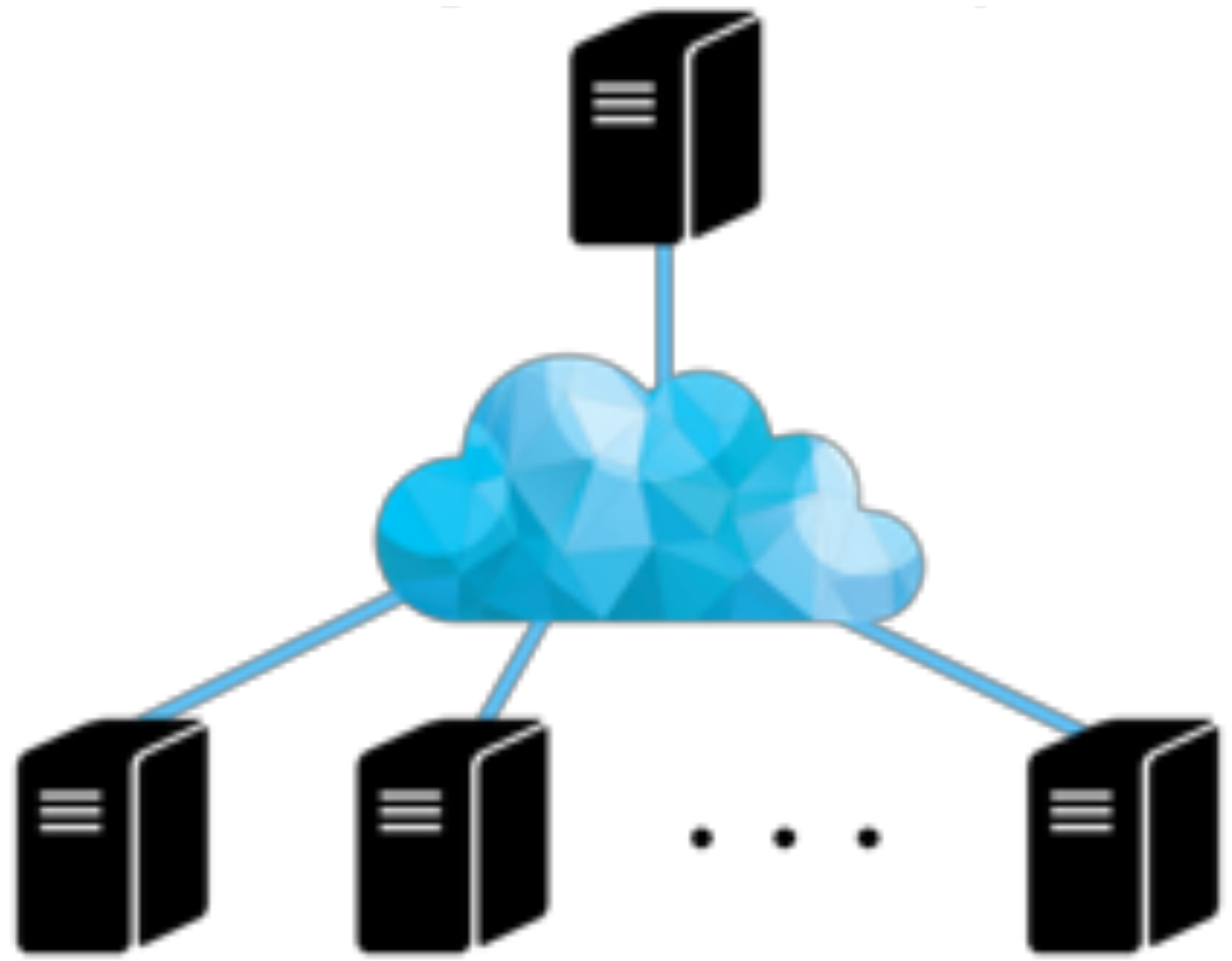

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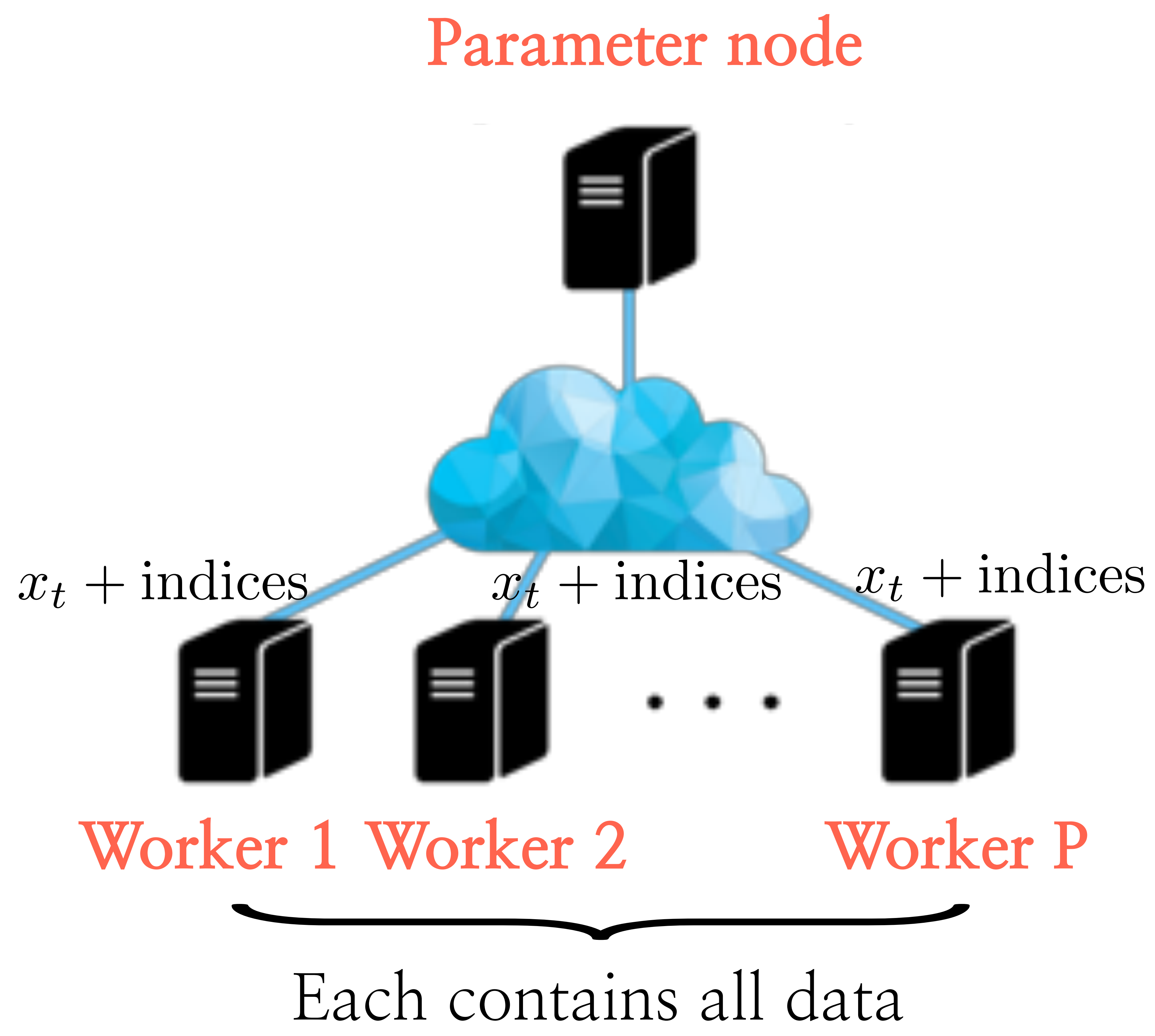


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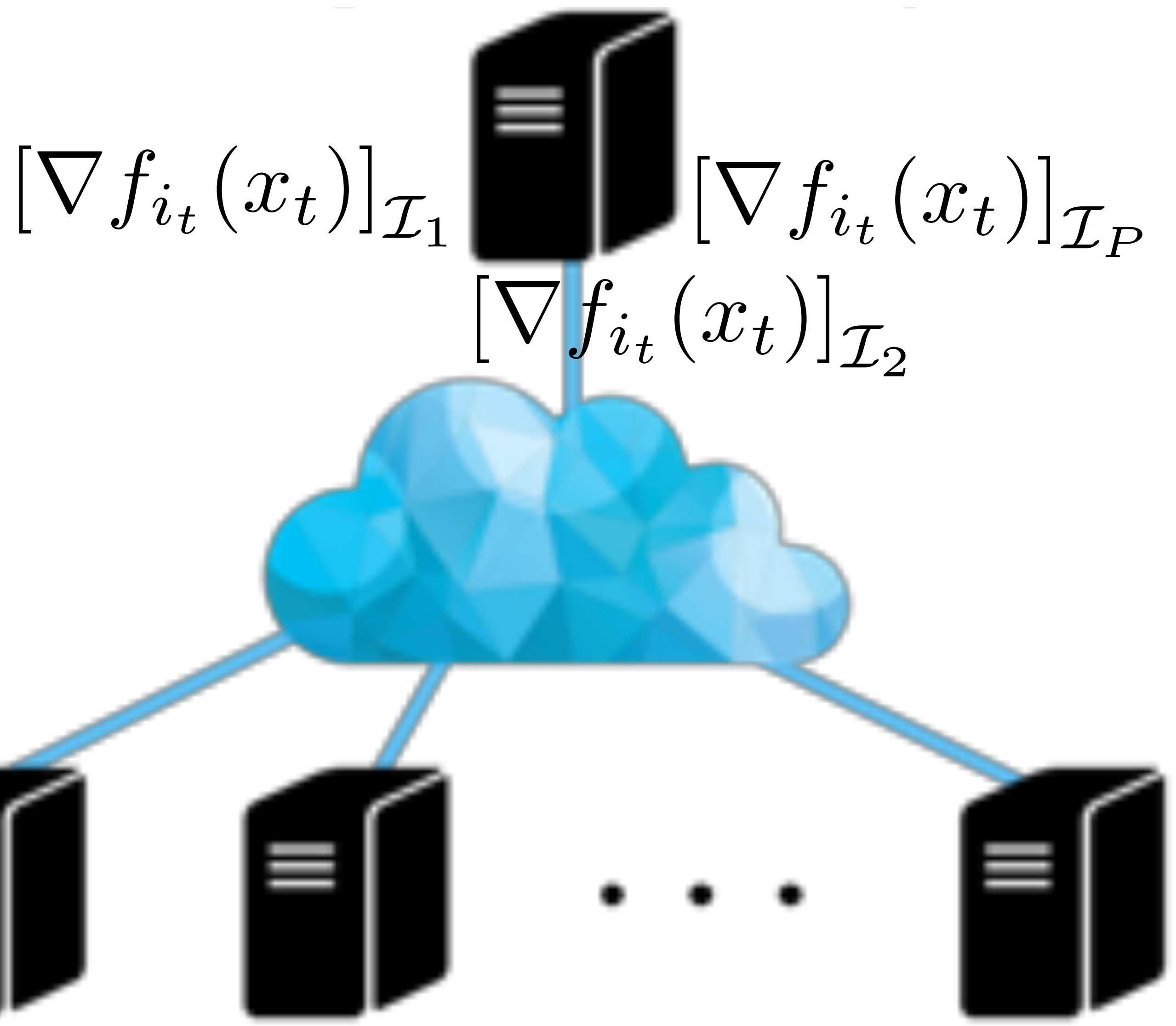


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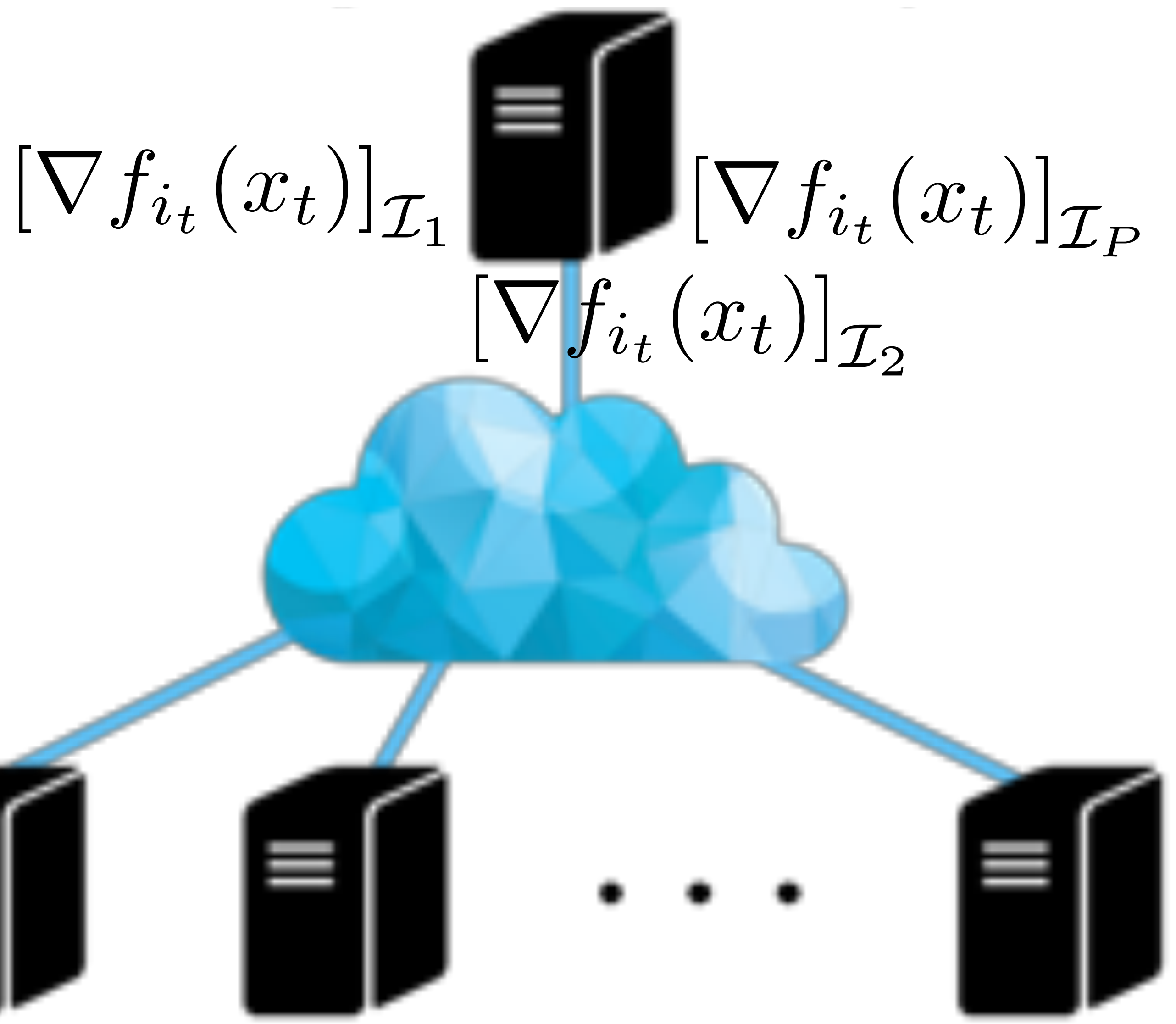


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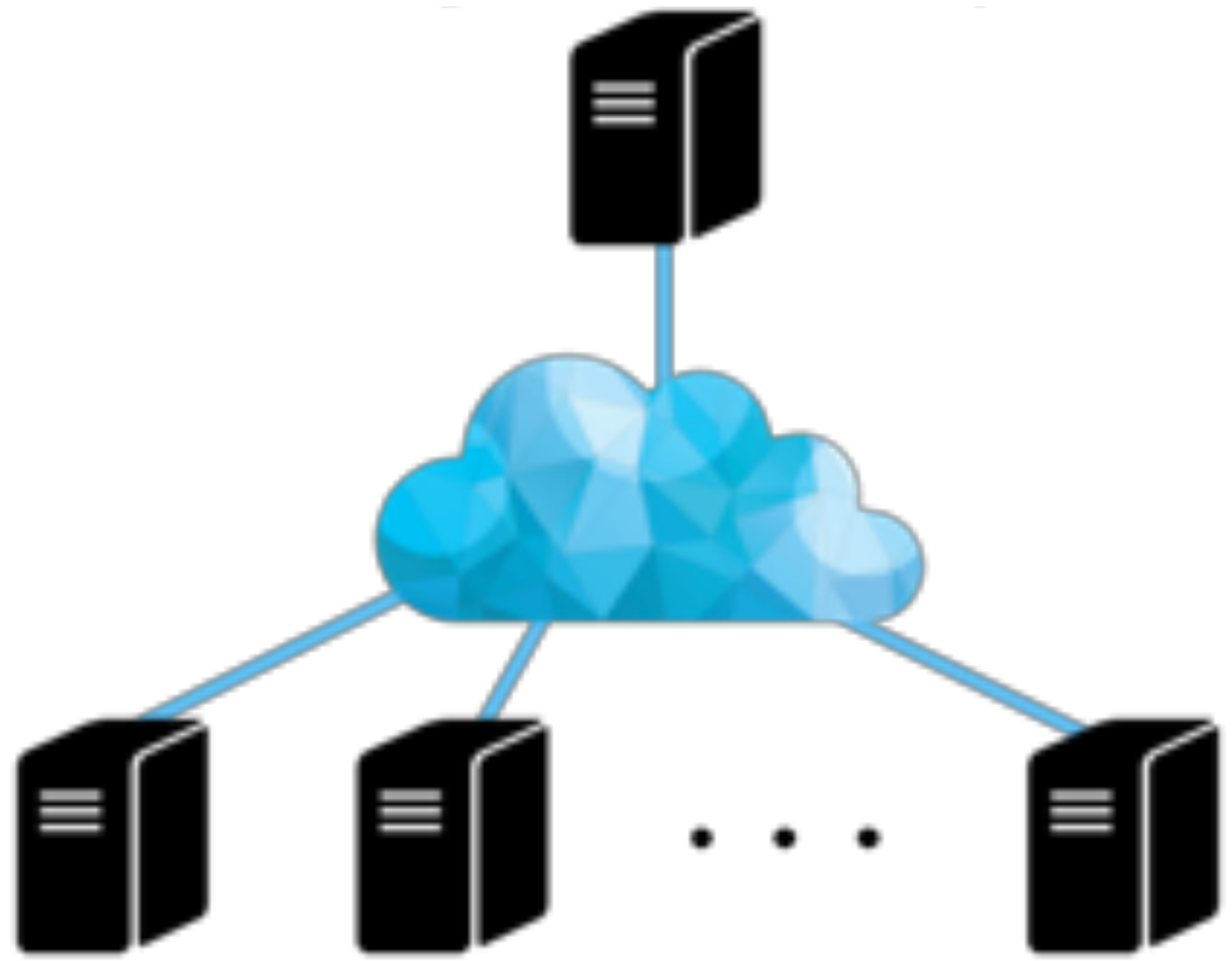


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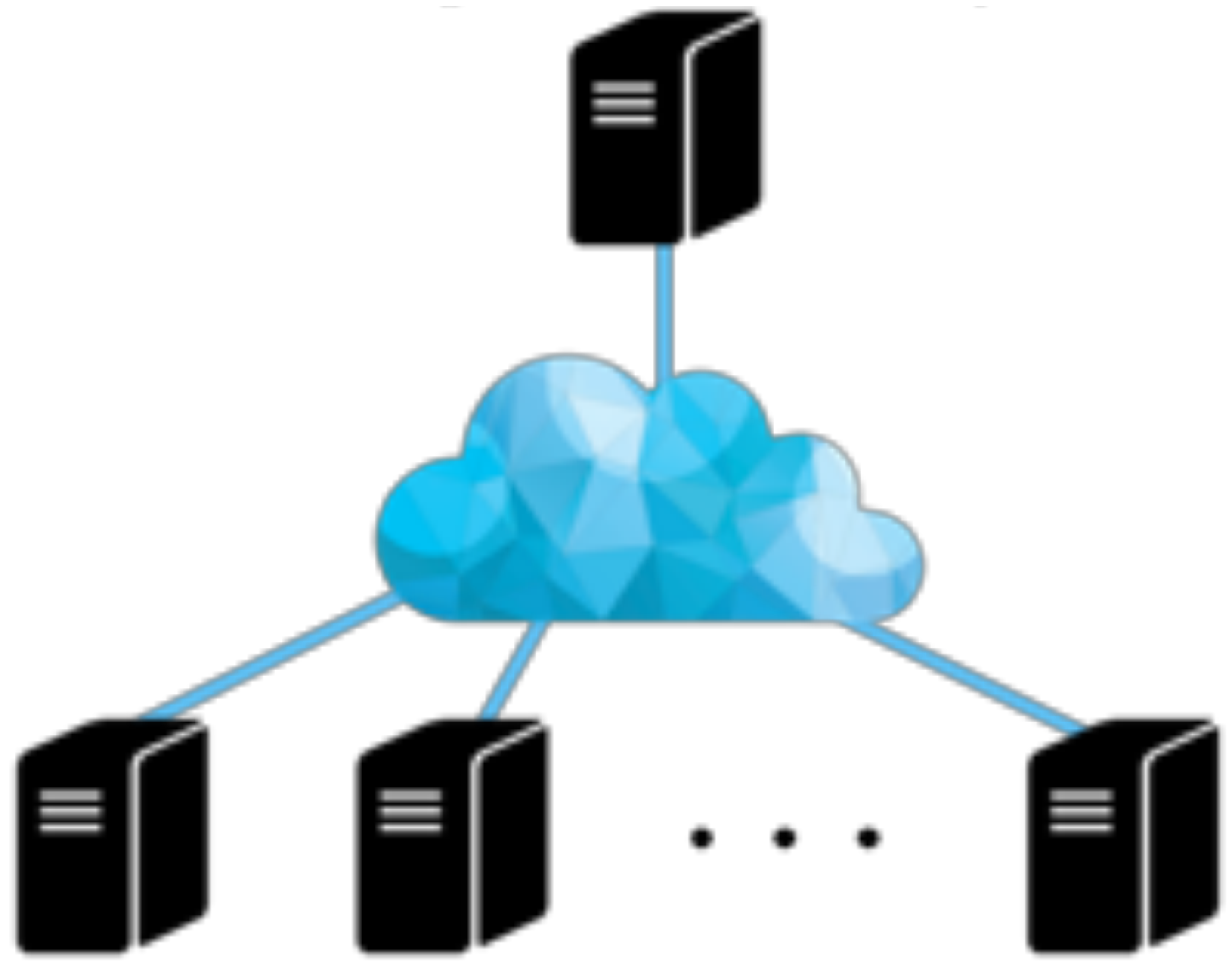

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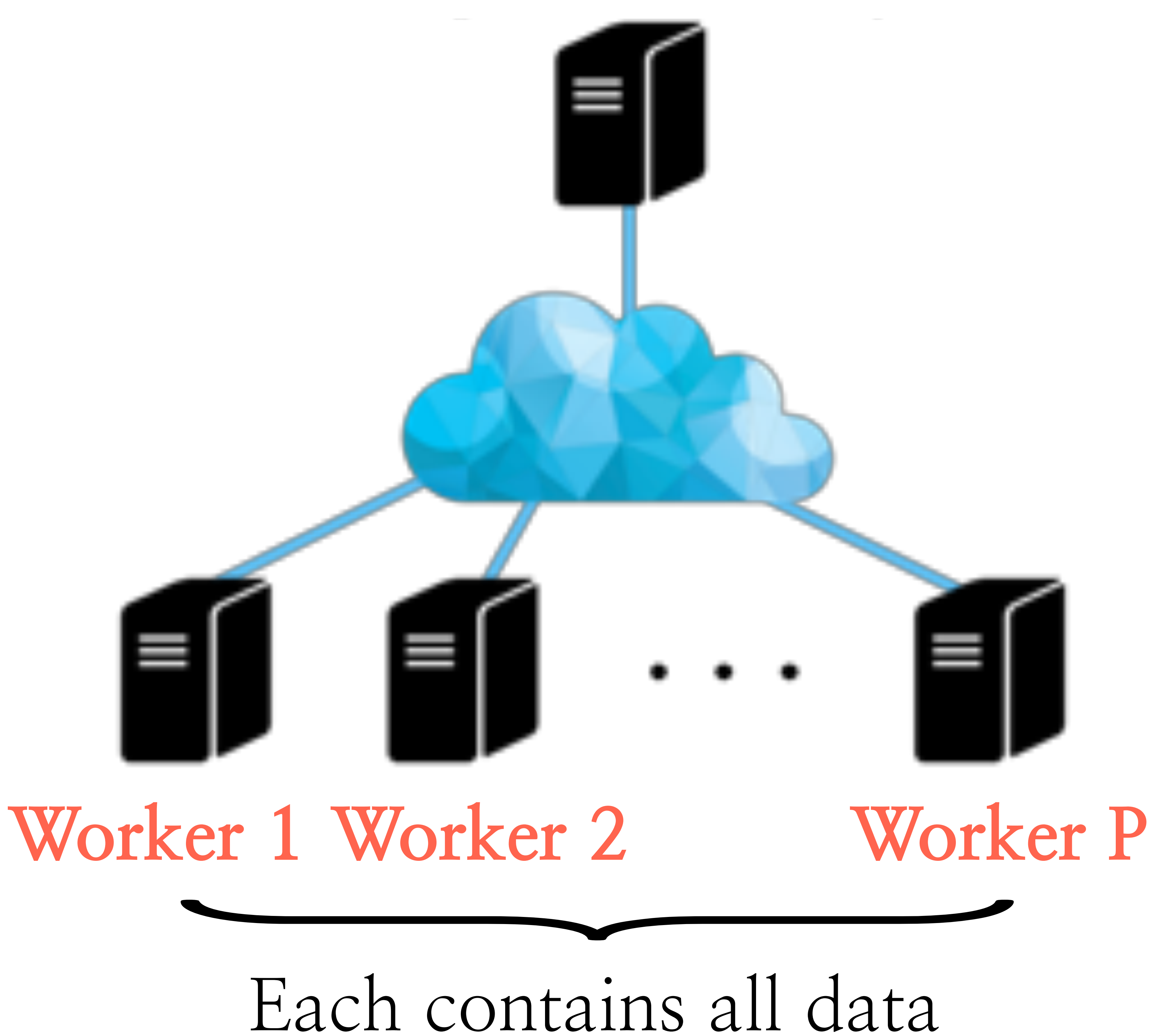


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*iii*) Could be an overkill to only compute updates for a subset of entries

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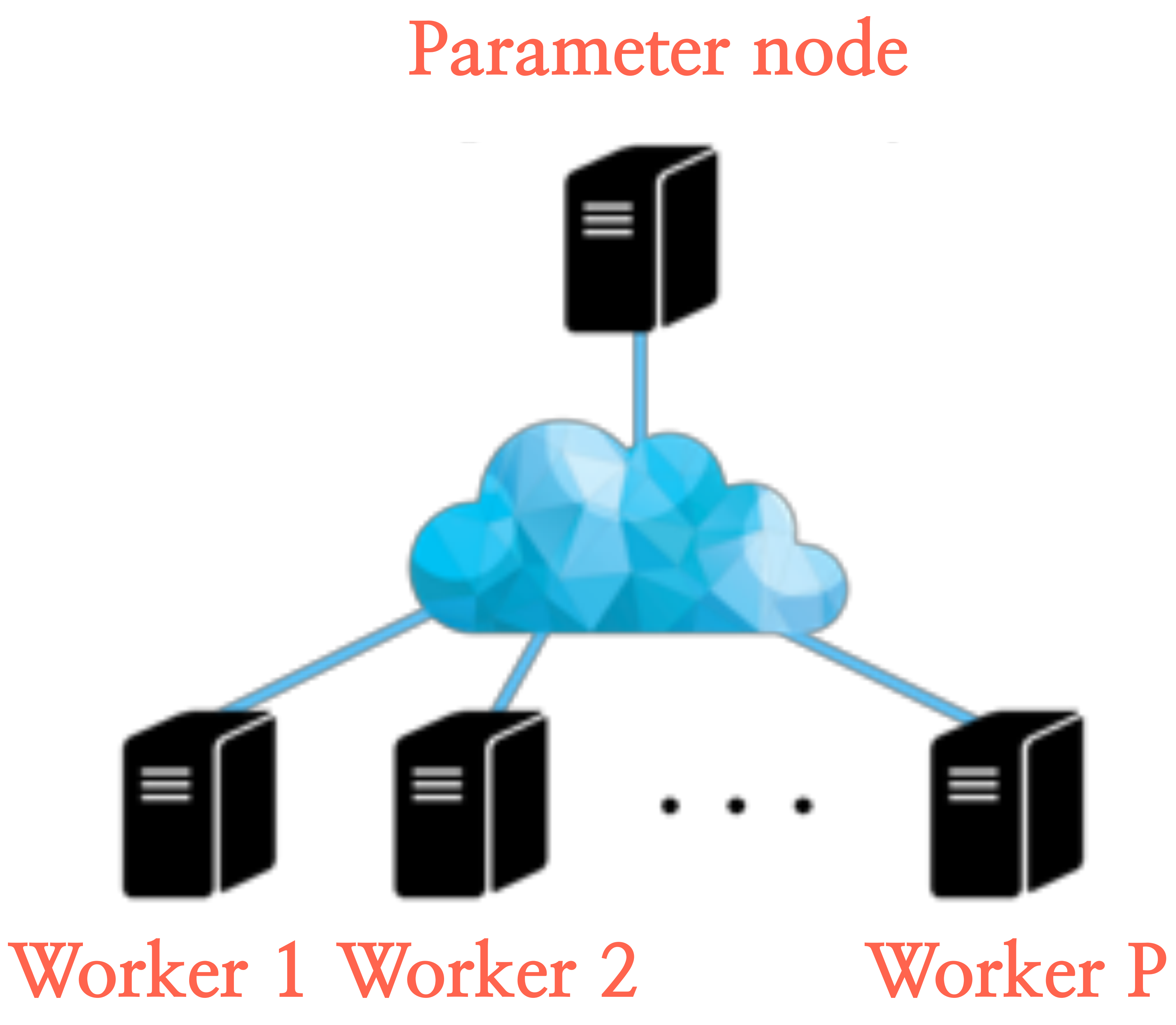


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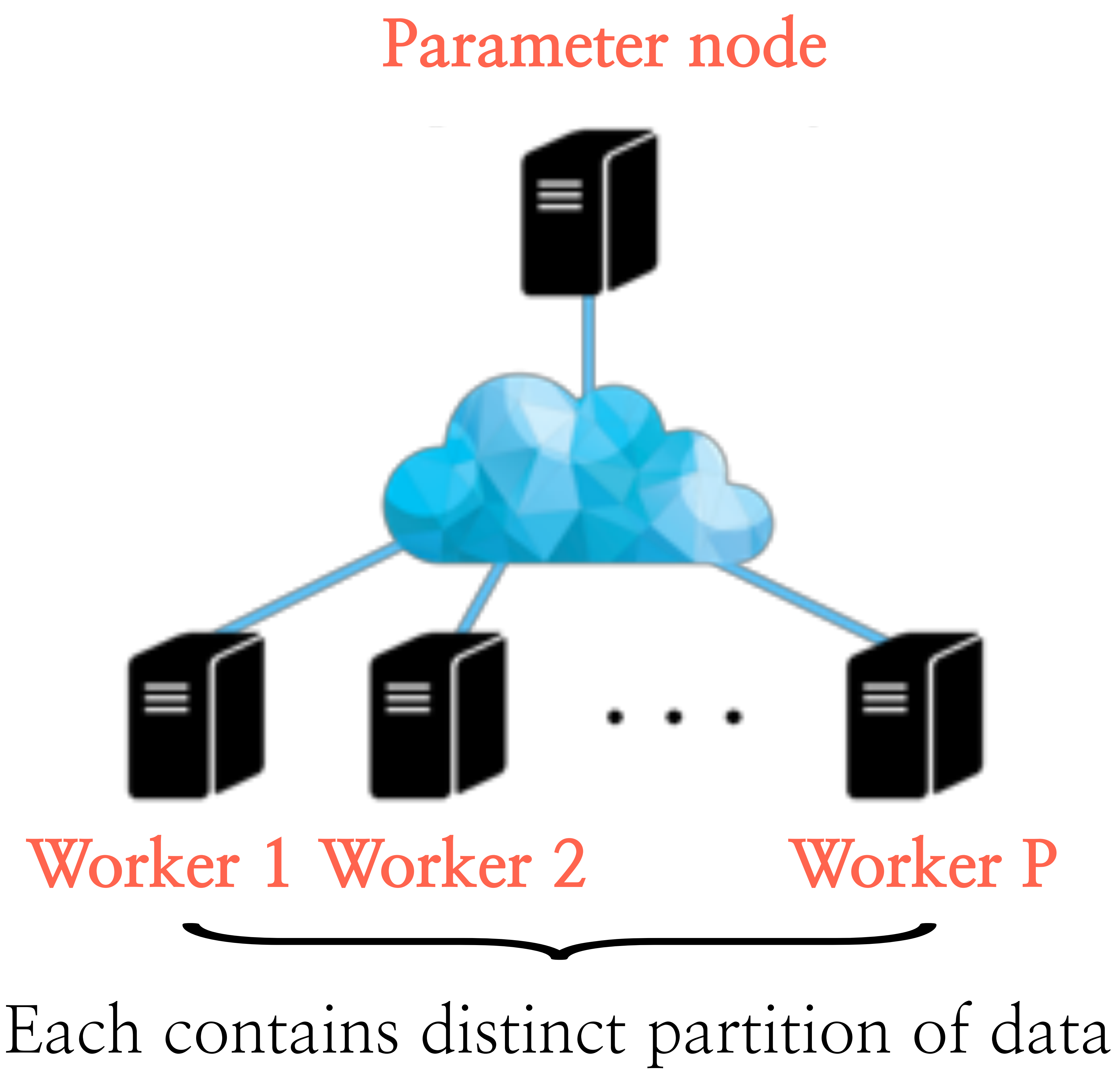


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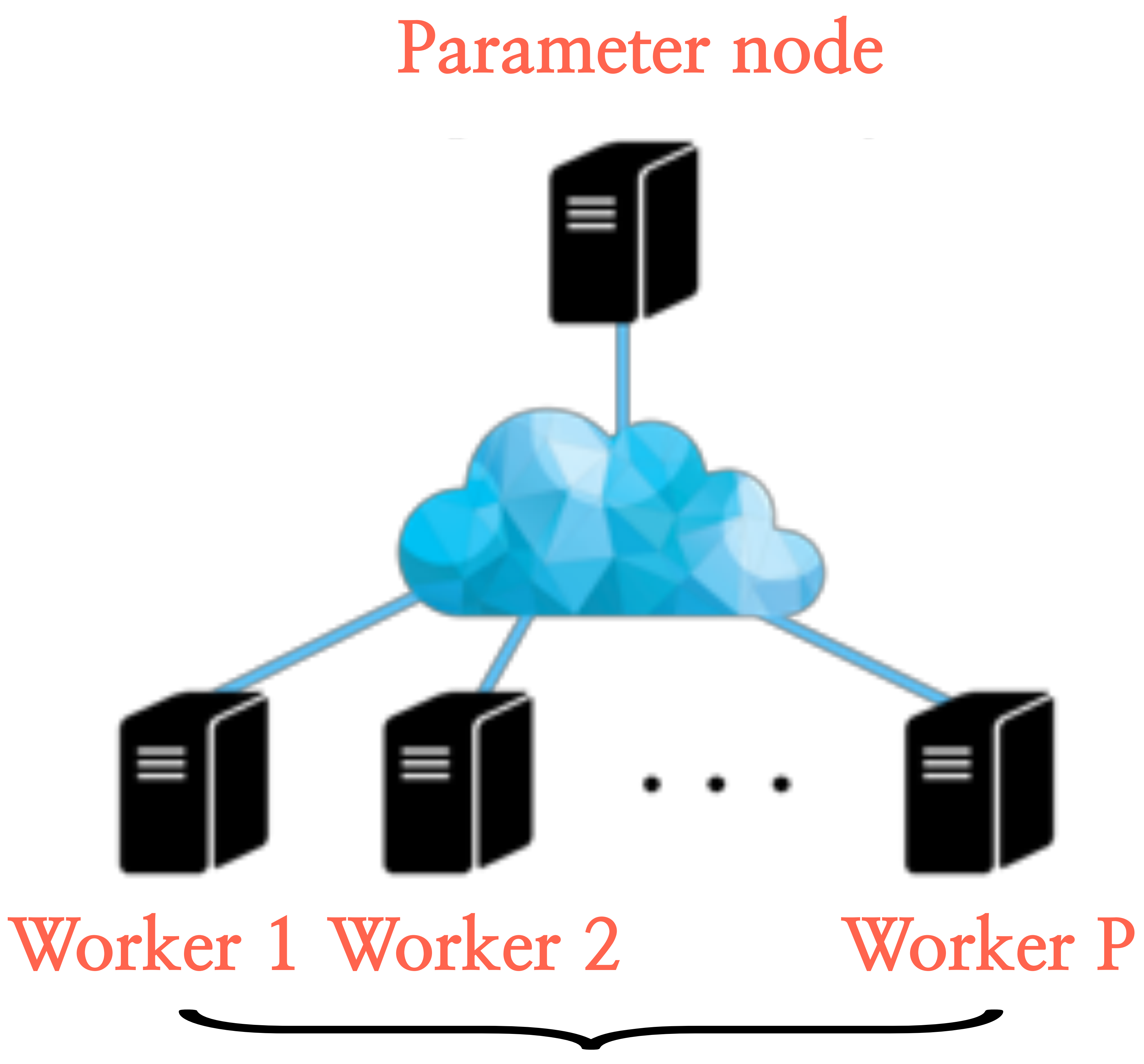


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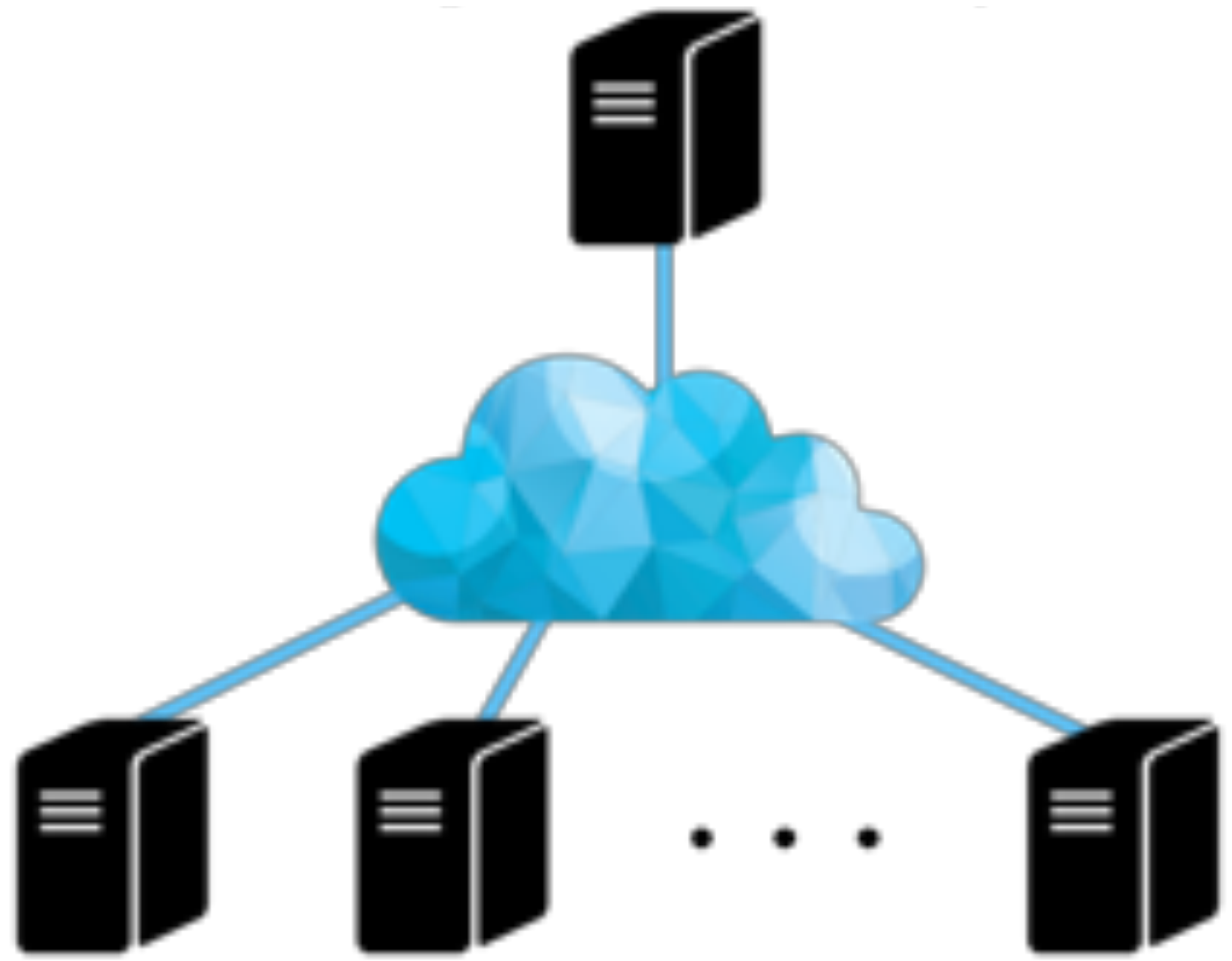


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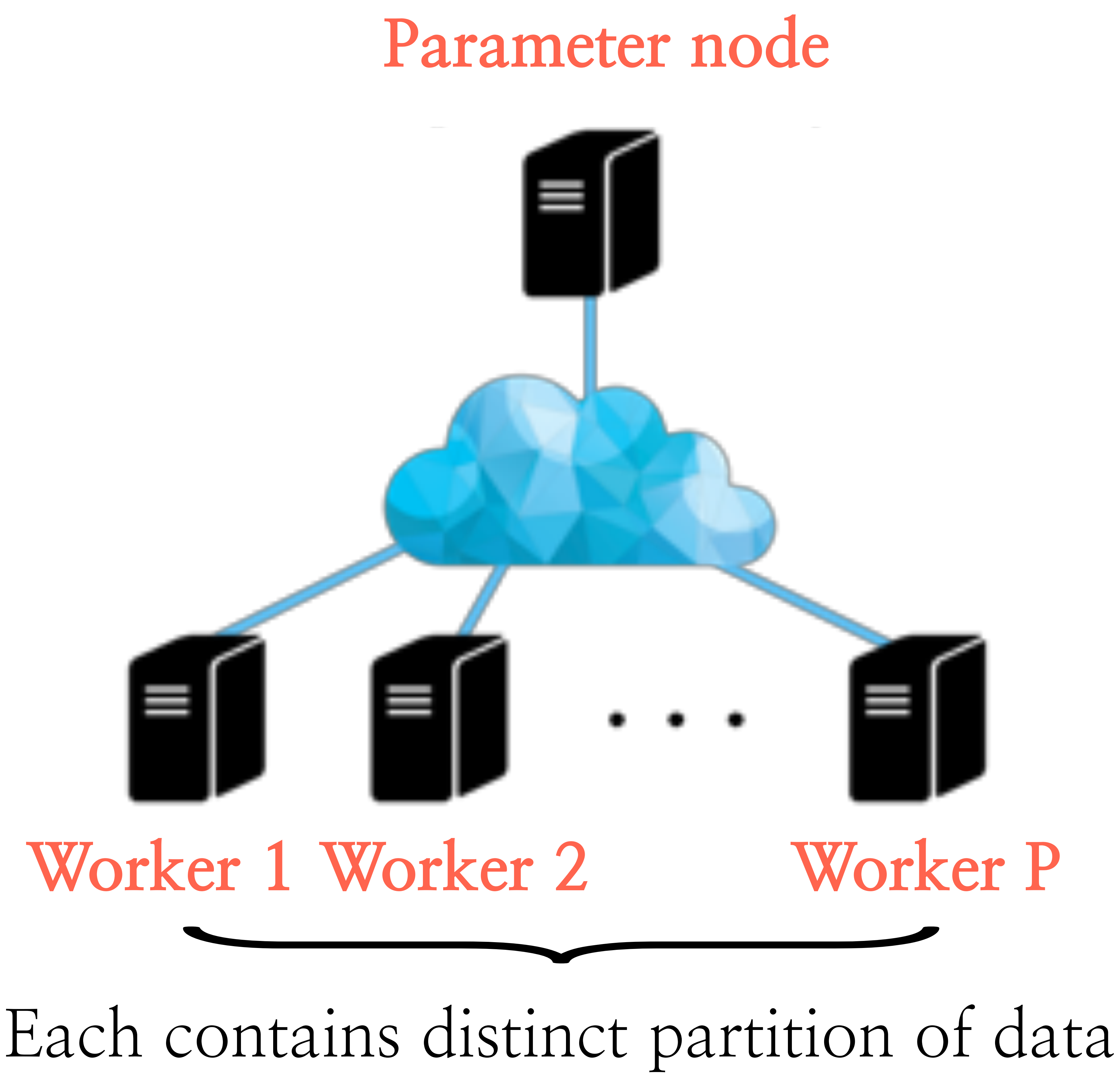


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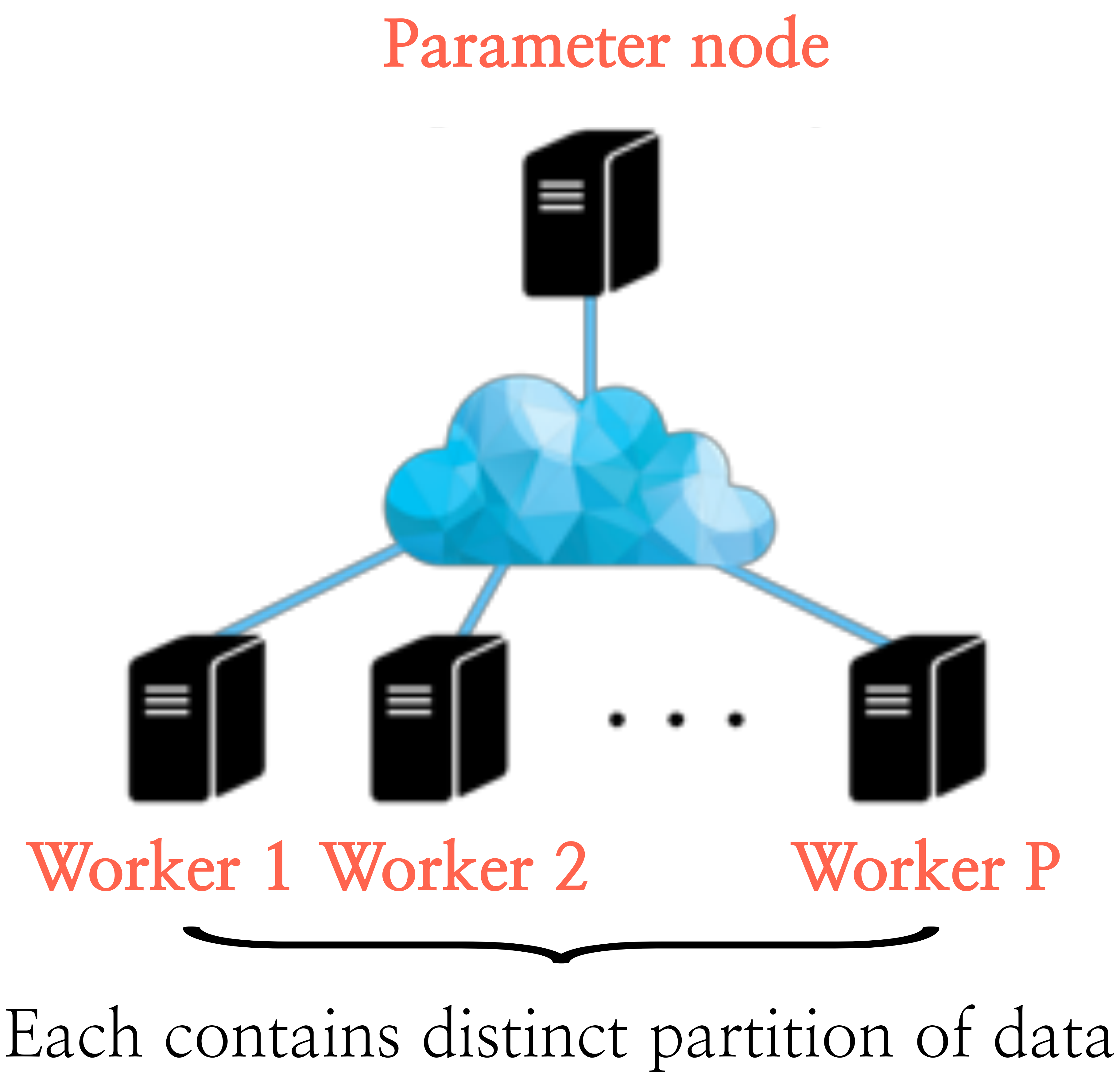


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*ii*) Introduces a tradeoff between statistical efficiency, computations efficiency (in terms of convergence) and communication efficiency

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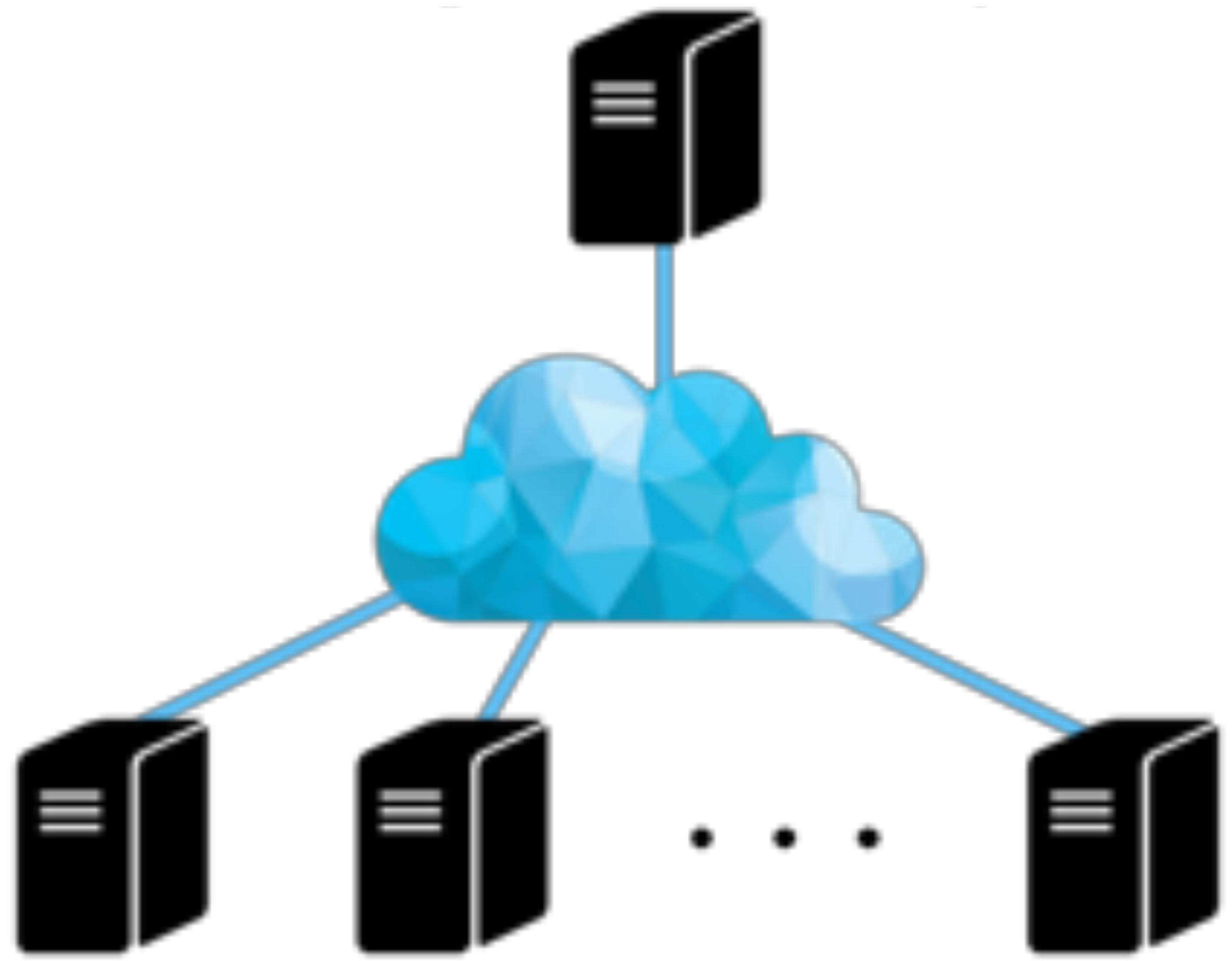

Worker 1 Worker 2 Worker P

Each contains distinct partition of data

*i*) Still requires synchronization; each worker has less work to do

*ii*) Introduces a tradeoff between statistical efficiency, computations efficiency (in terms of convergence) and communication efficiency

*iii*) Usually computing $\nabla f_{i_t}(x_t)$ is cheap per node

(Discussion about large batch training)

# Distributing gradient computations

- What if we run **mini-batch SGD** in parallel and combine at the end:

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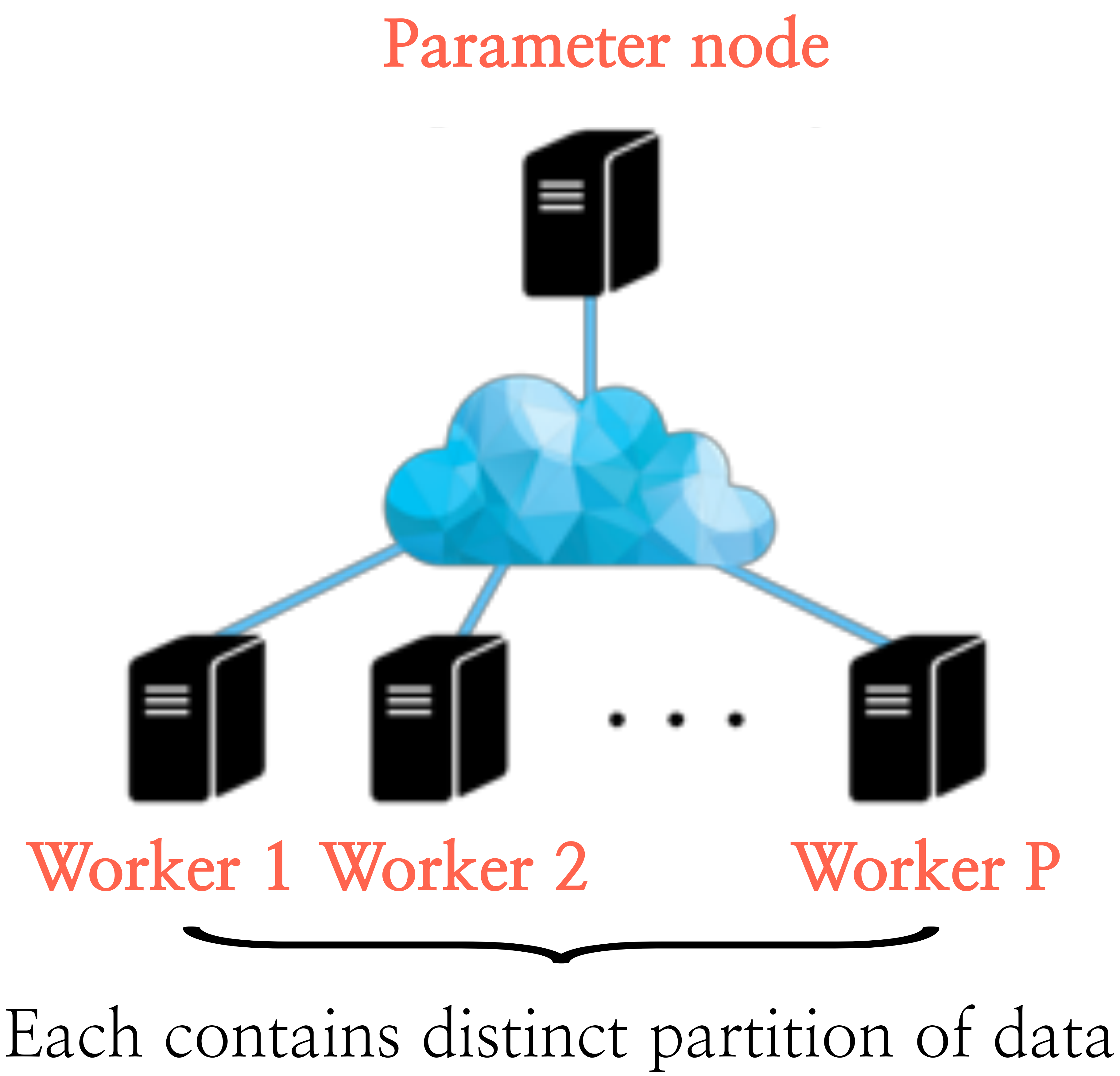

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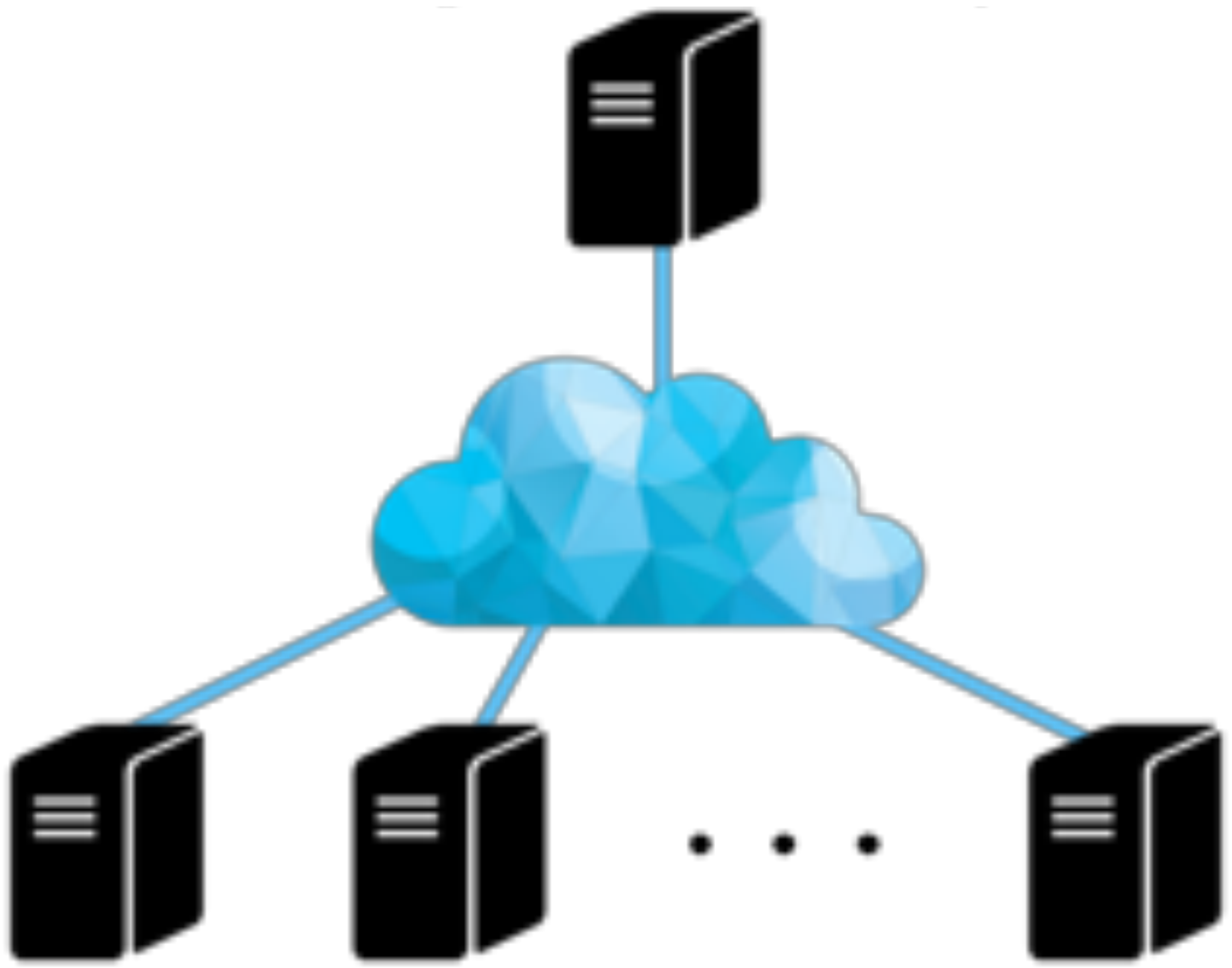

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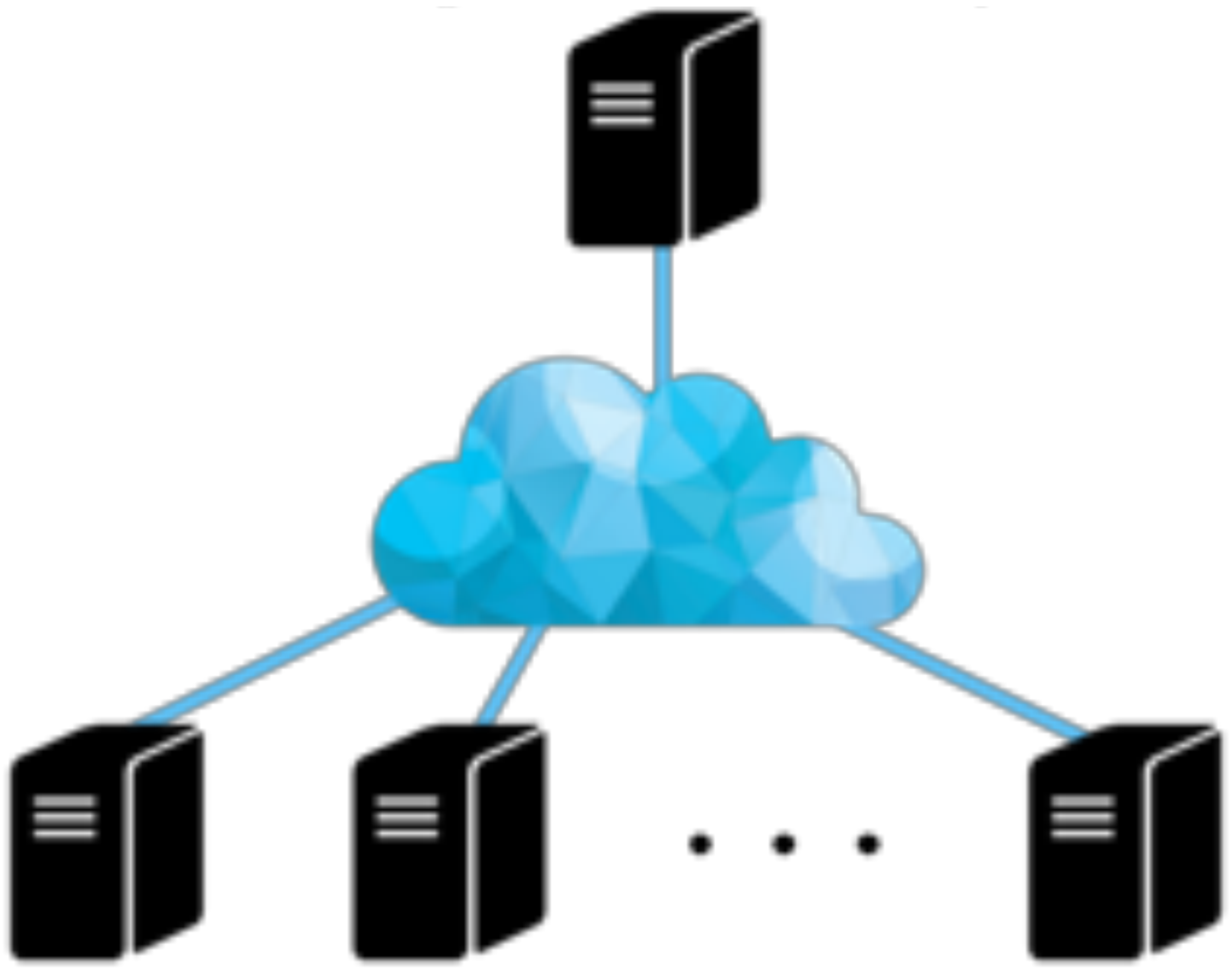

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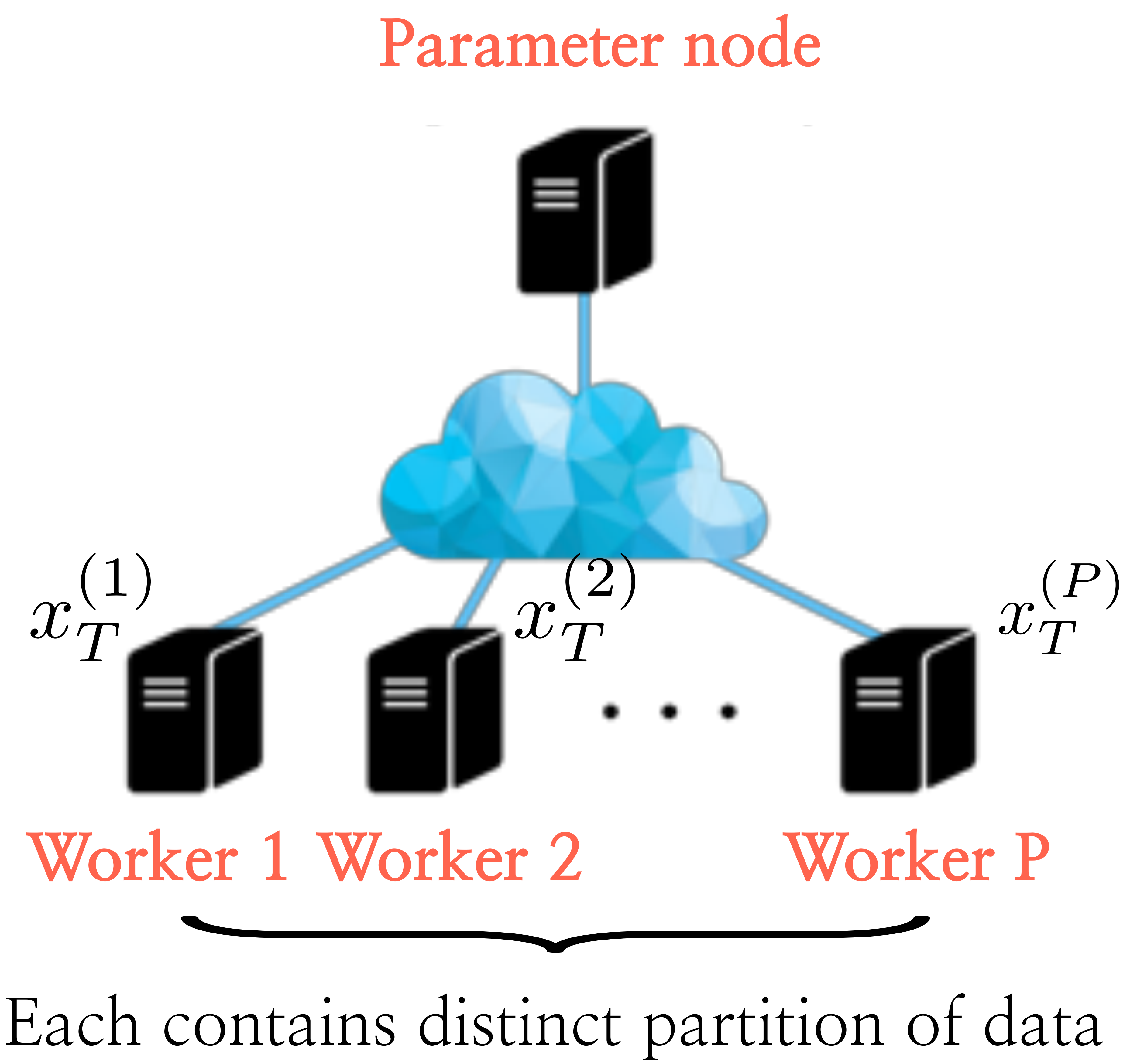


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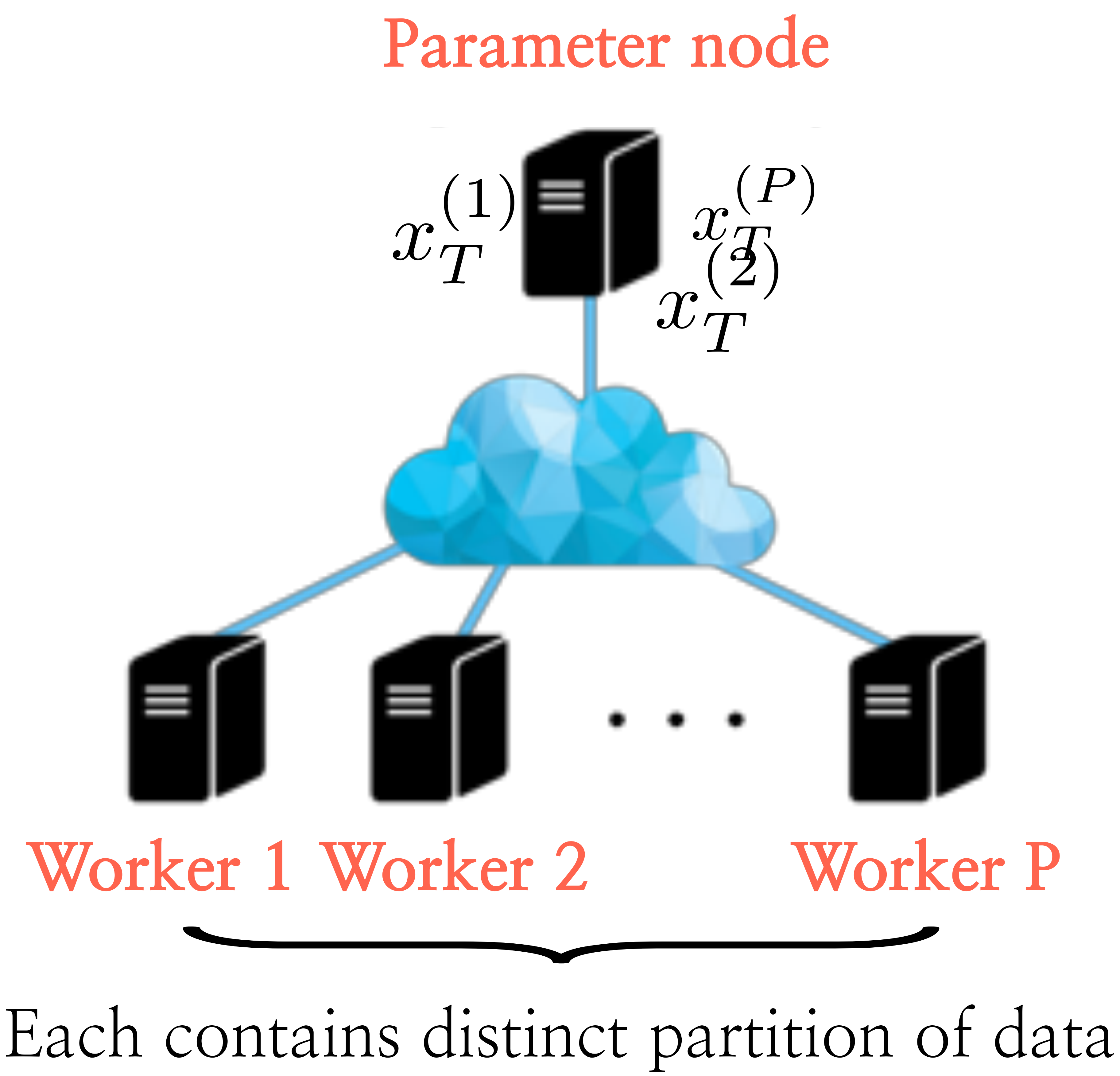


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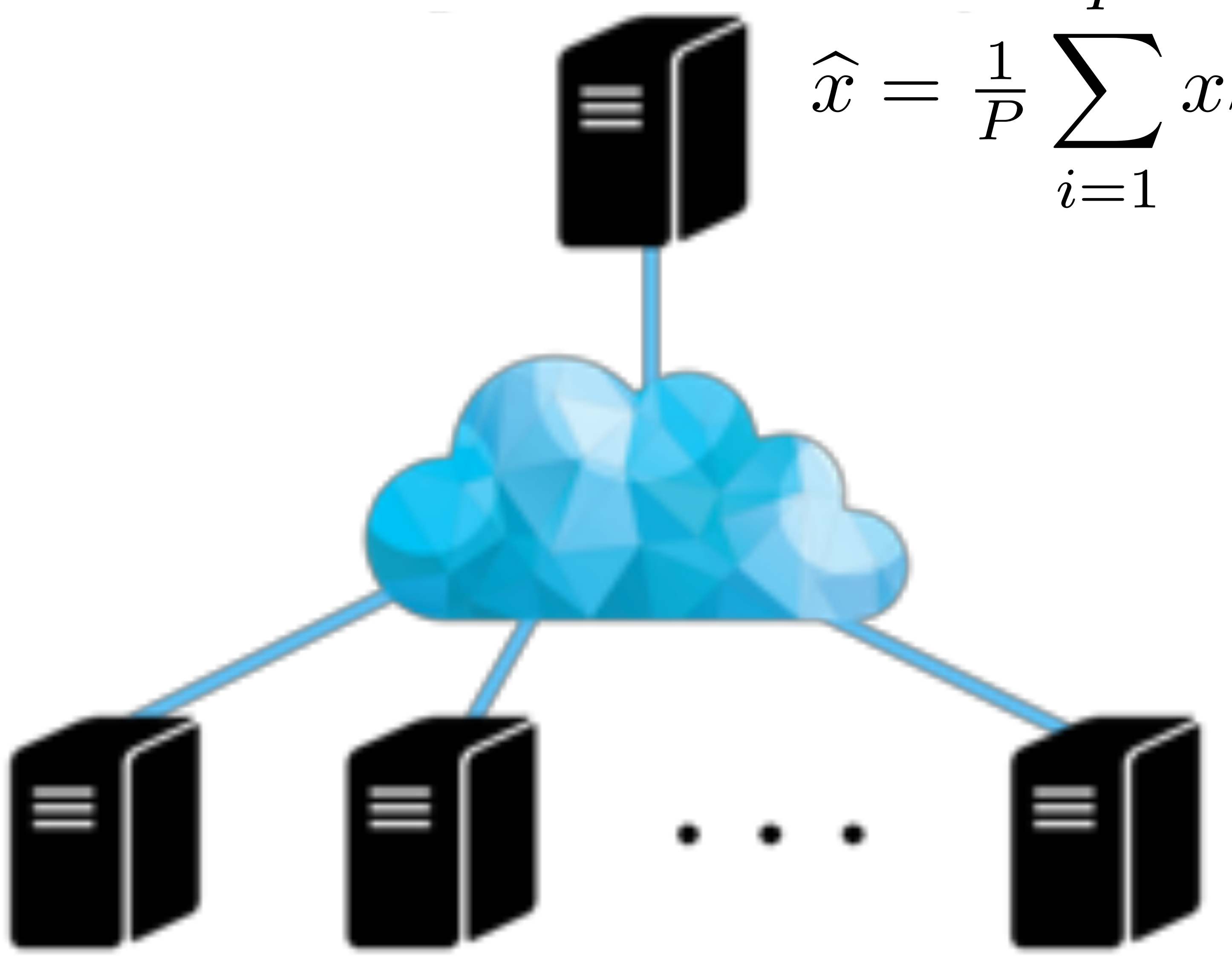
Parameter node

$$\widehat{x} = \frac{1}{P} \sum_{i=1}^{P} x_T^{(i)}$$



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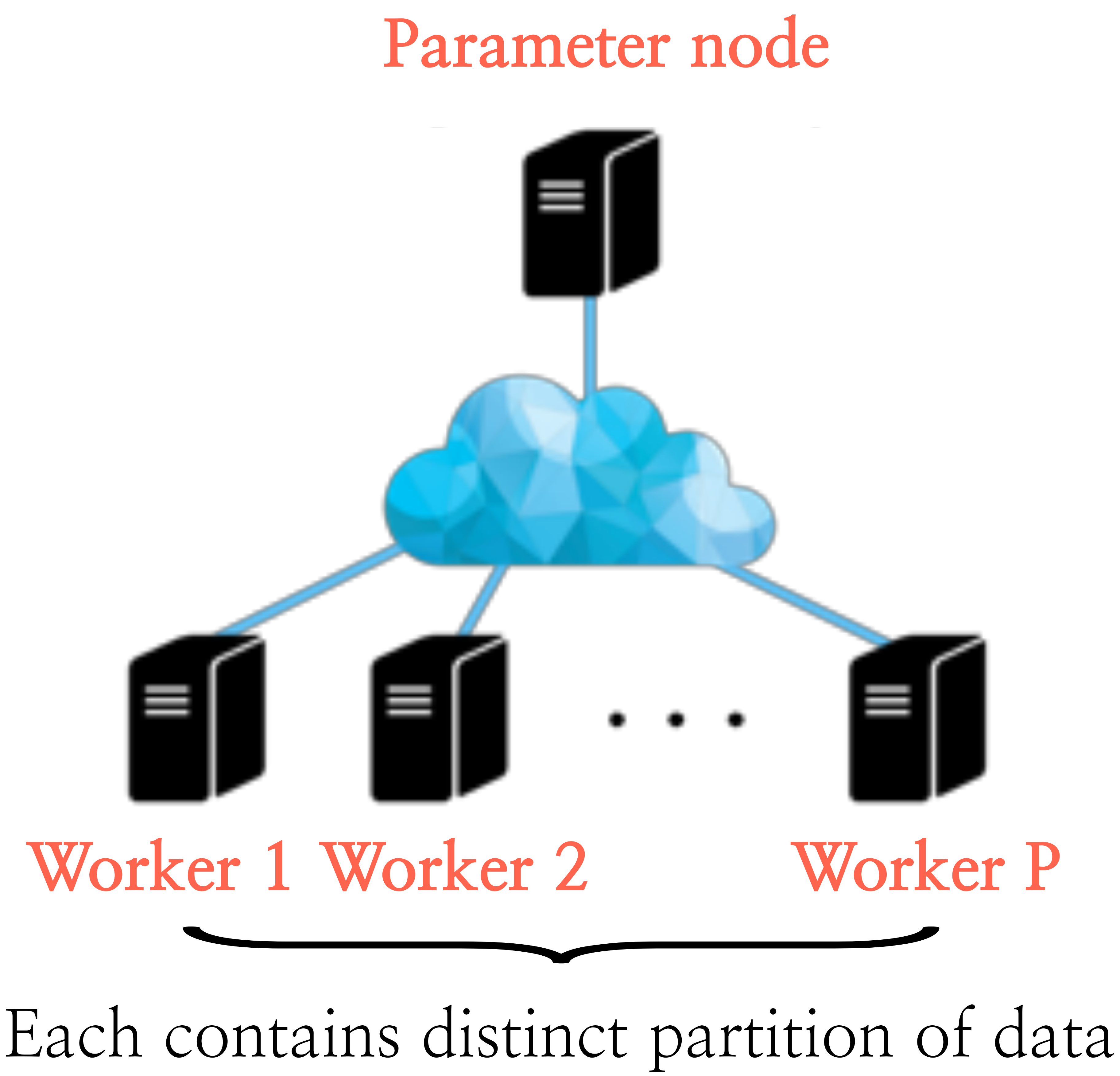


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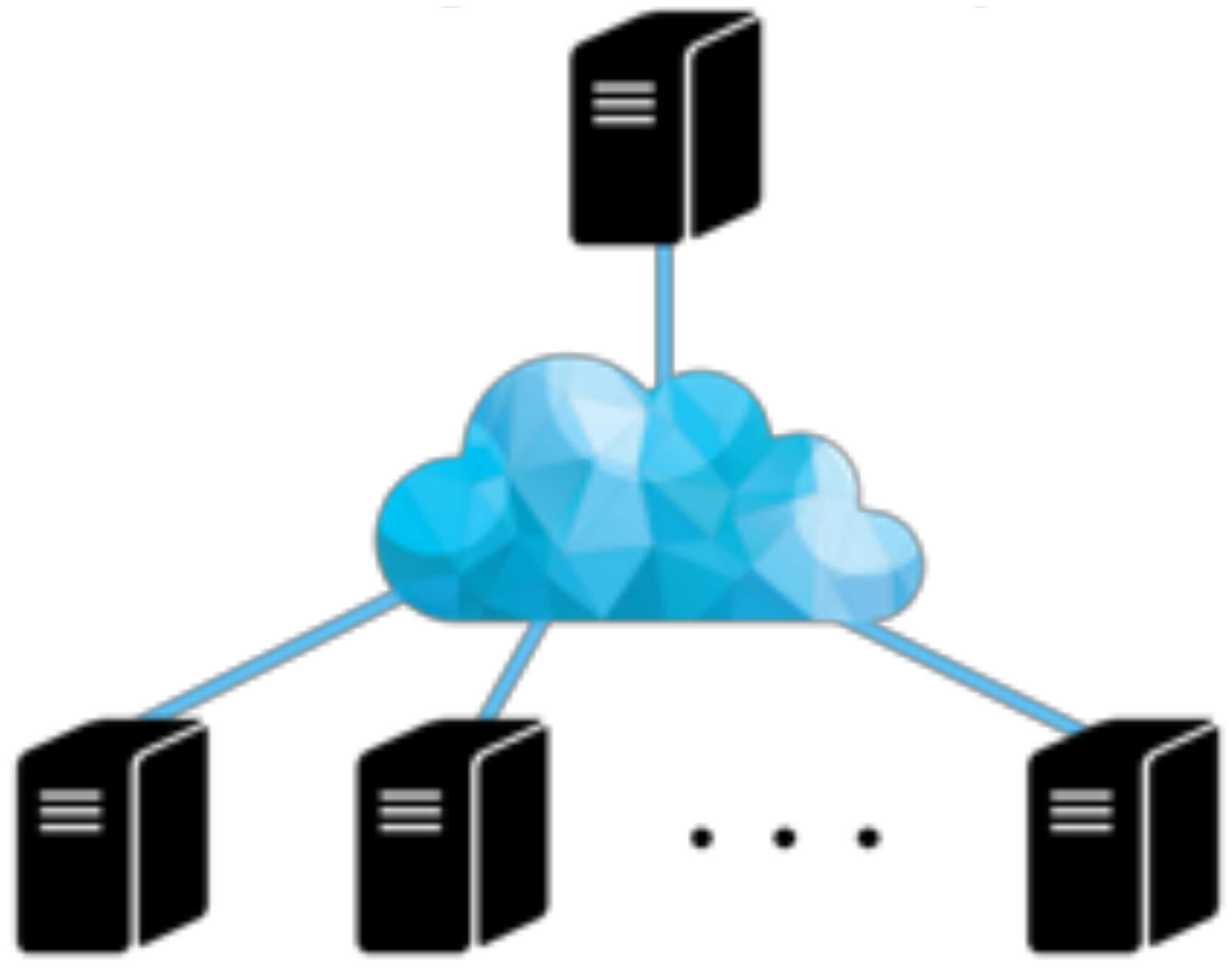

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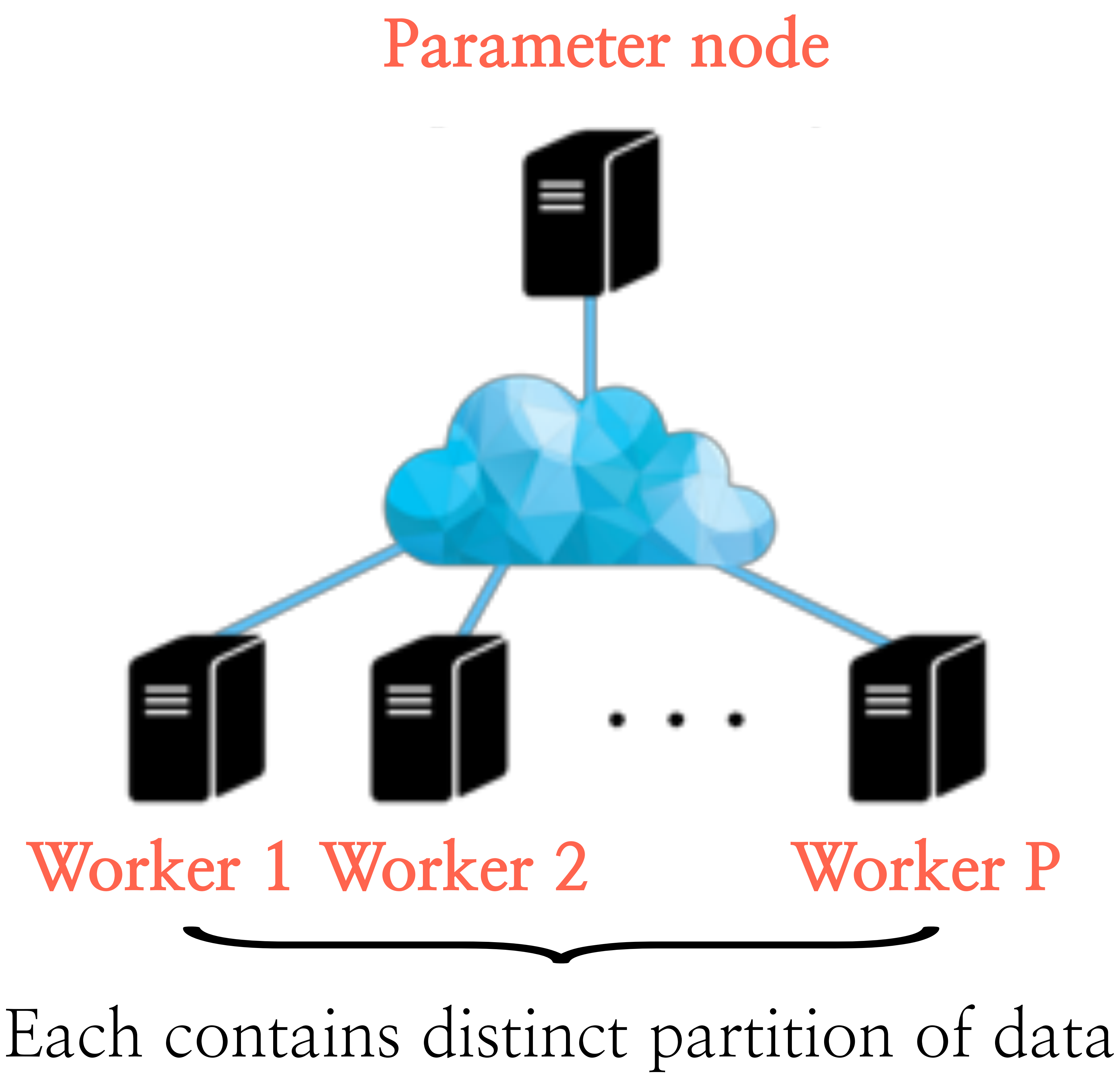


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$iii)$ Final decision is prediction averaging – similar ideas hold for random forests

# Using distributed computing in a different way

- Run code in parallel as a way for hyperparameter optimization

$$x_{t+1} = x_t - \eta_1 \sum_{i \in \mathcal{I}_t} \nabla f_i(x_t) \qquad \bullet \quad \bullet \quad \bullet \qquad x_{t+1} = x_t - \eta_q \sum_{i \in \mathcal{I}_t} \nabla f_i(x_t)$$

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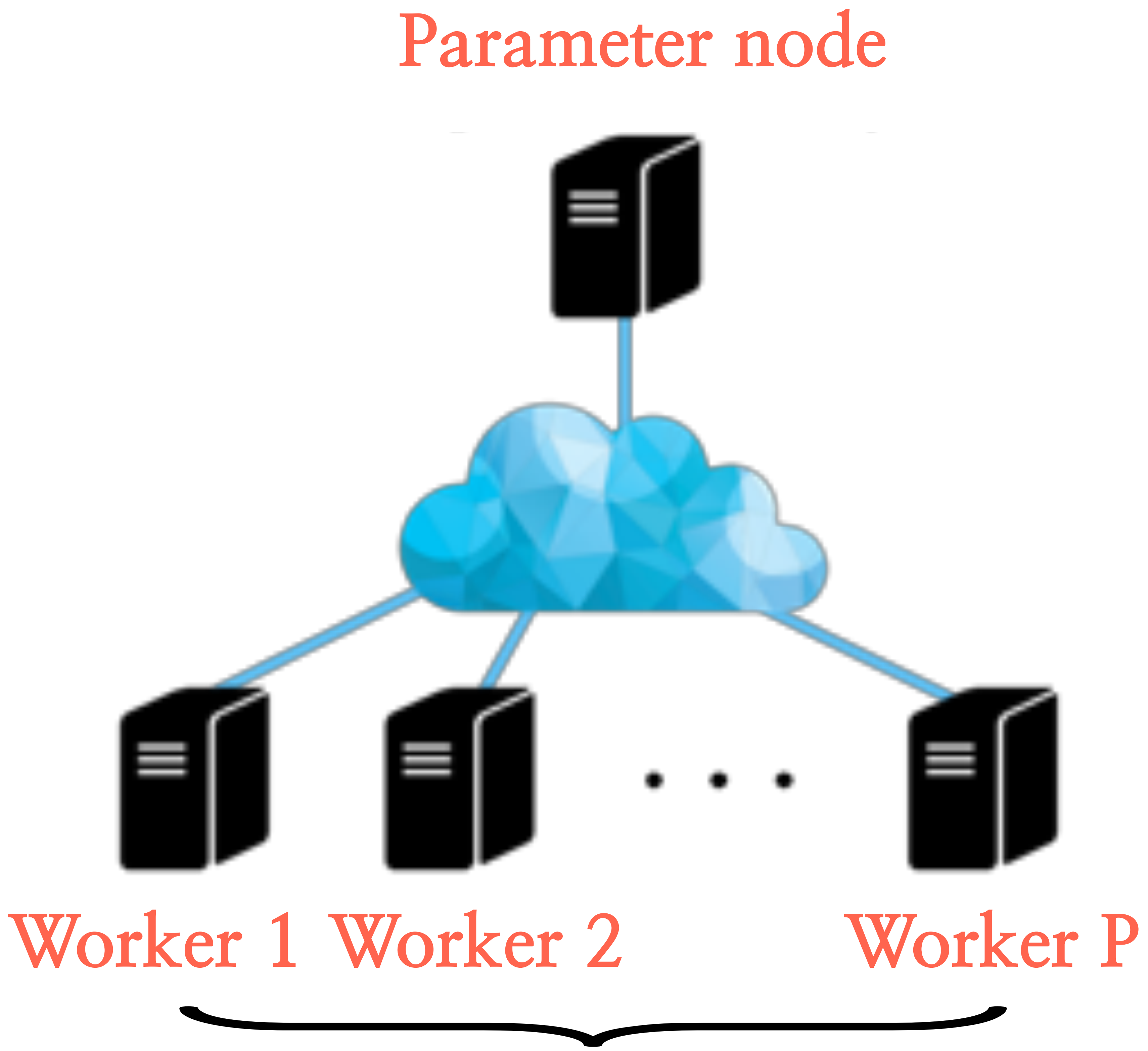
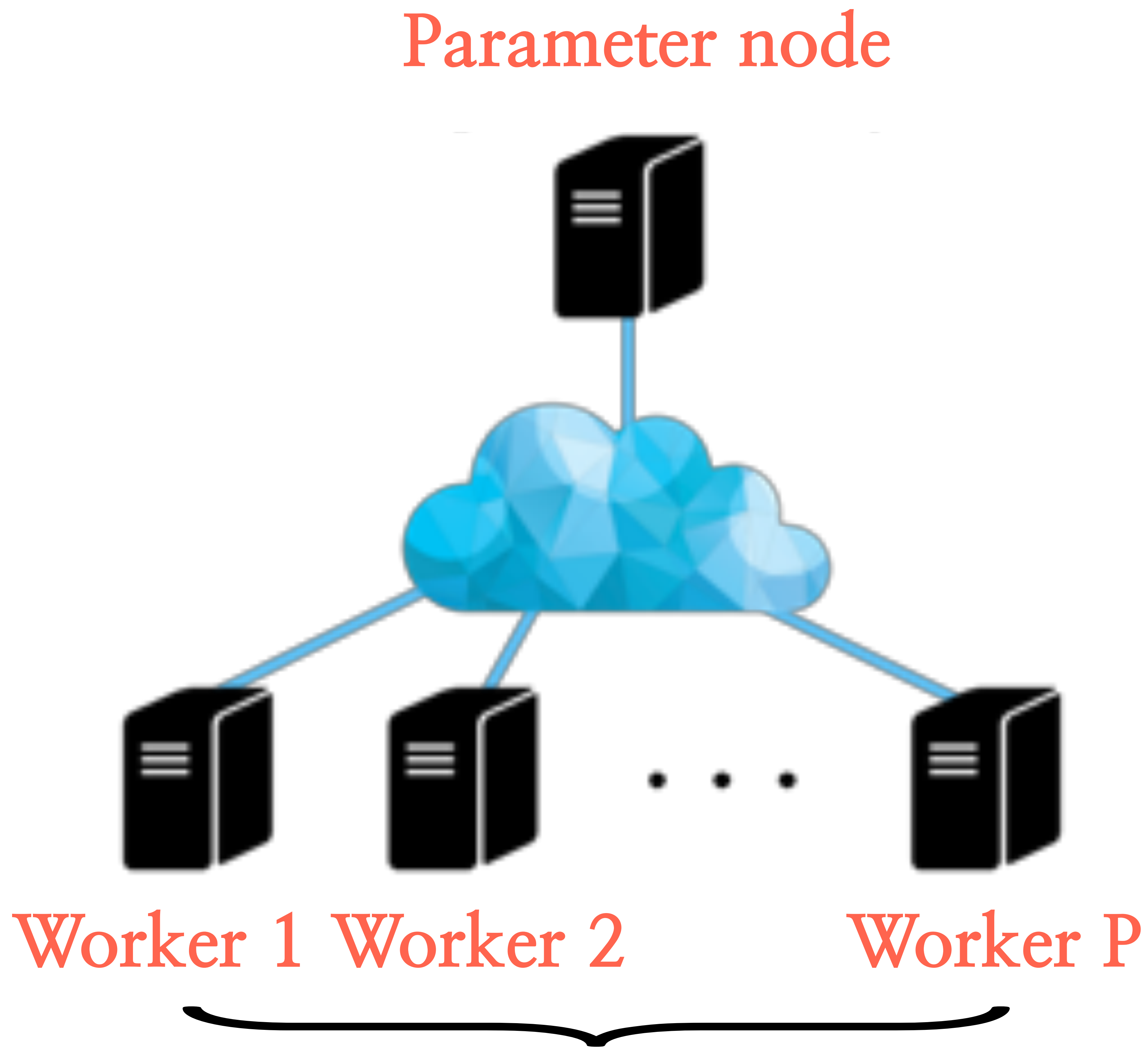



Each contains distinct partition of data

• • •

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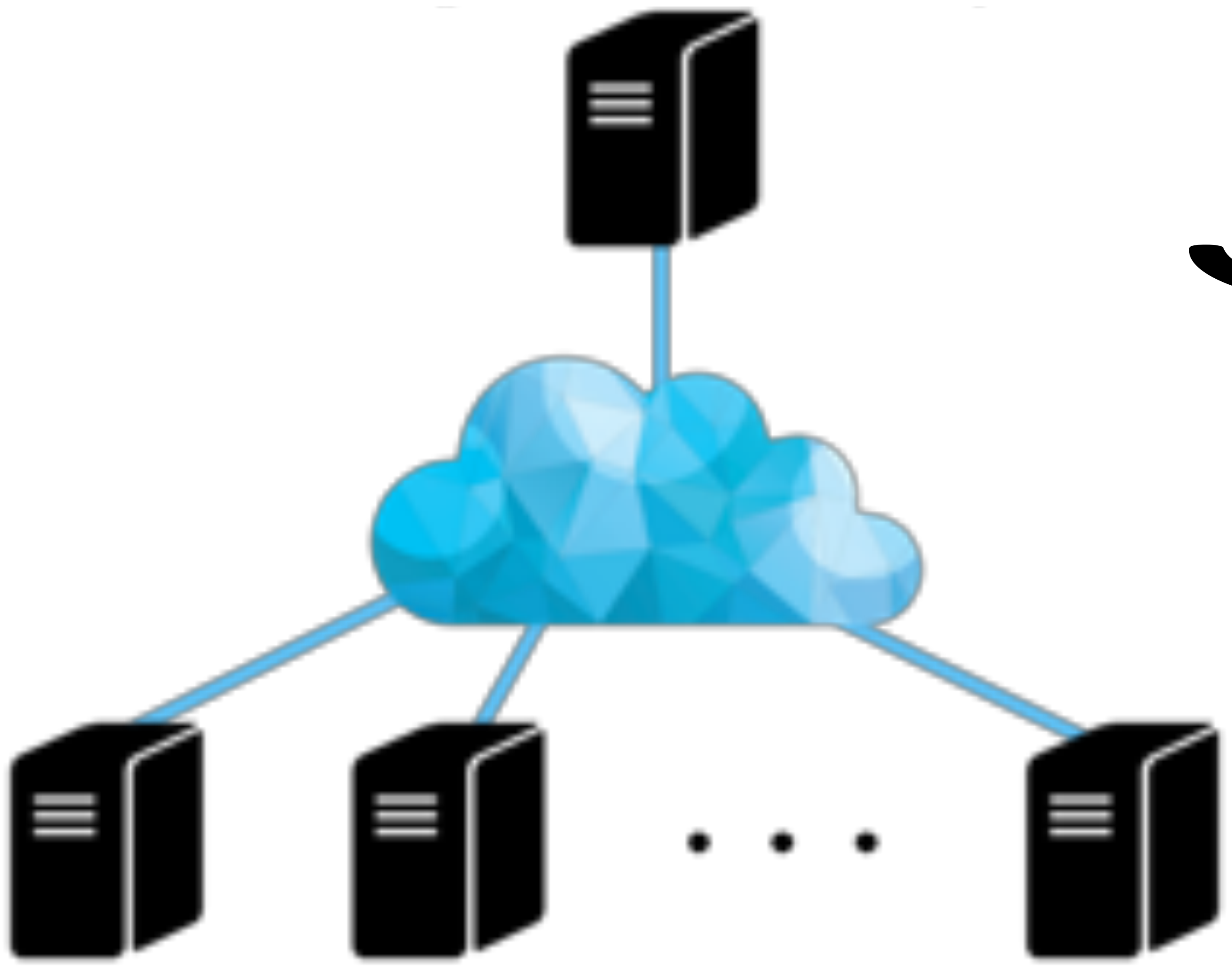

Worker 1 Worker 2 Worker P

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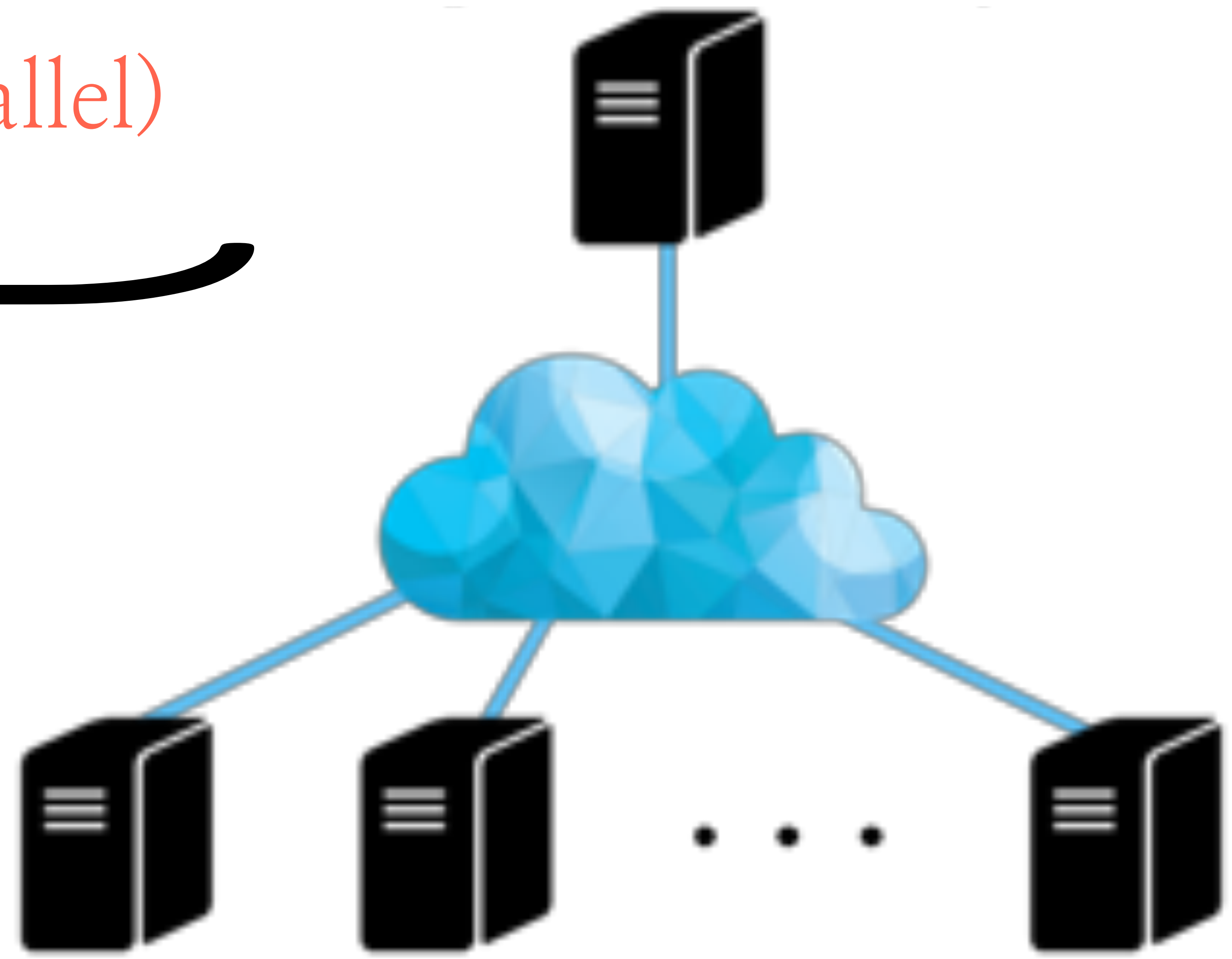
$\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}_{}$ (q different settings, ran in parallel)

$\cdots$

Parameter node



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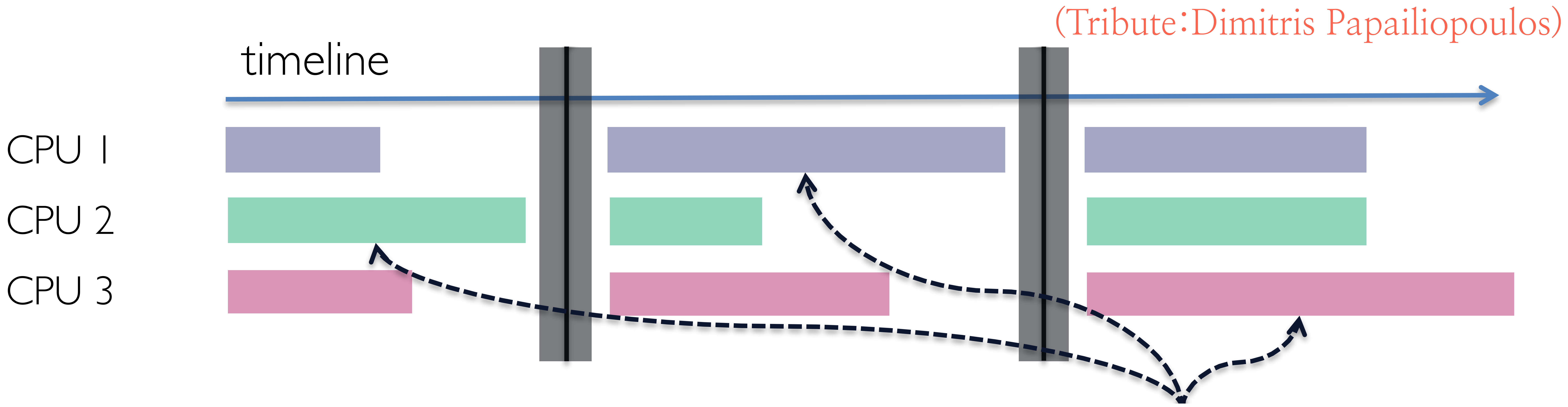
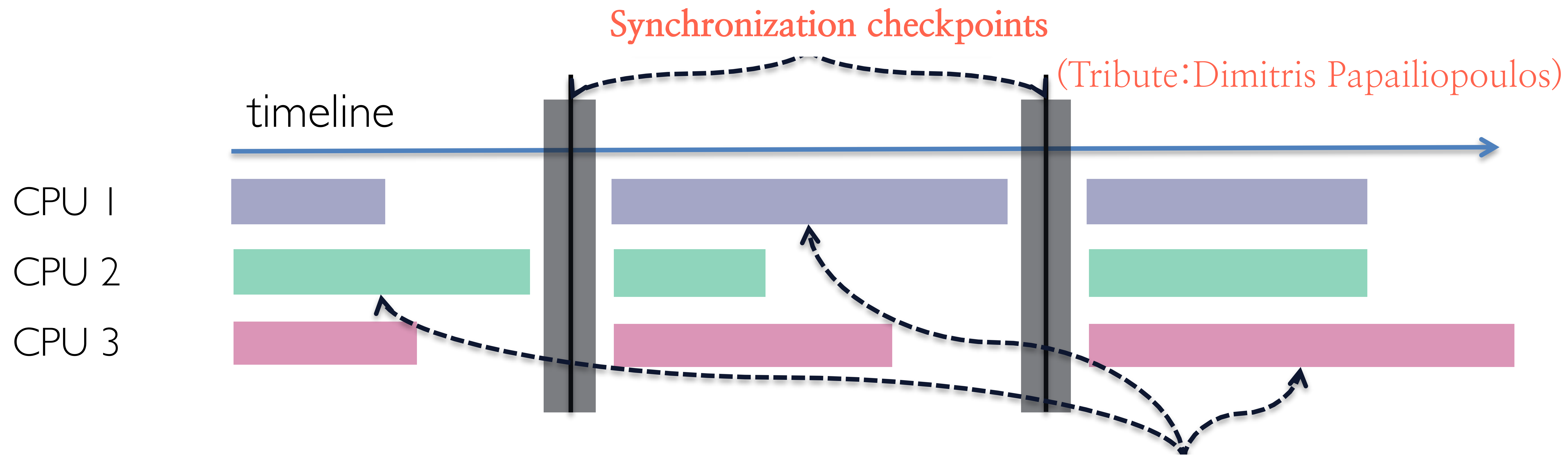
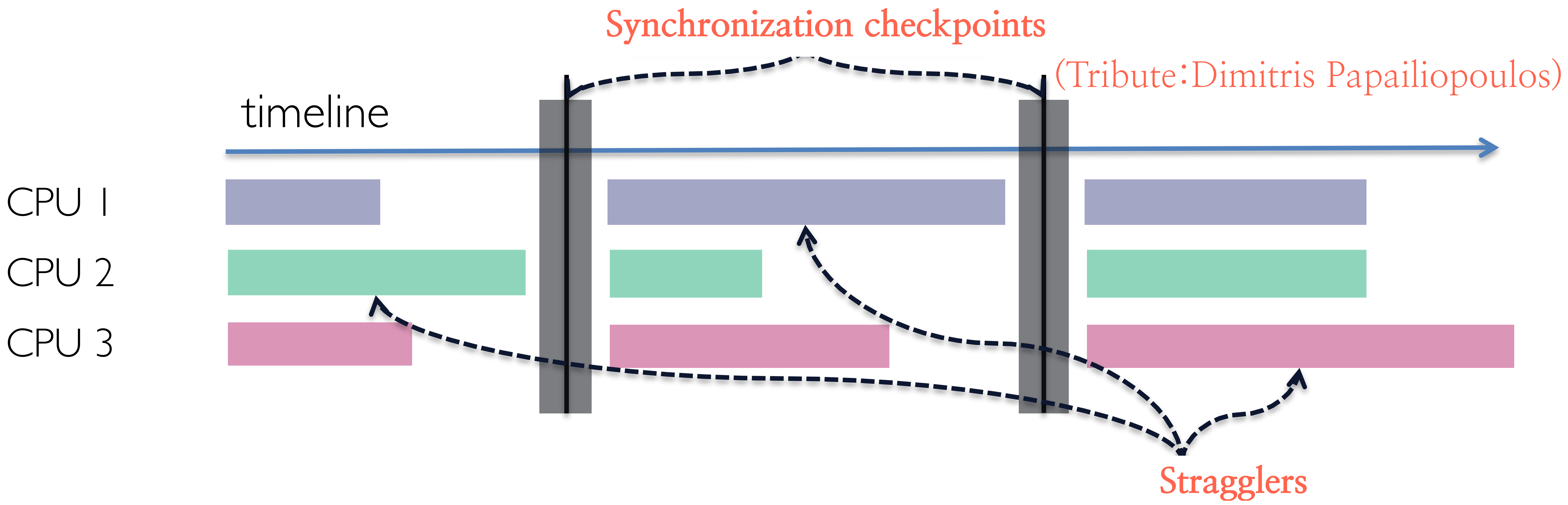

(Tribute:Dimitris Papailiopoulos)
timeline
CPU 1
CPU 2
CPU 3



Synchronization checkpoints
(Tribute:Dimitris Papailiopoulos)
timeline
CPU 1
CPU 2
CPU 3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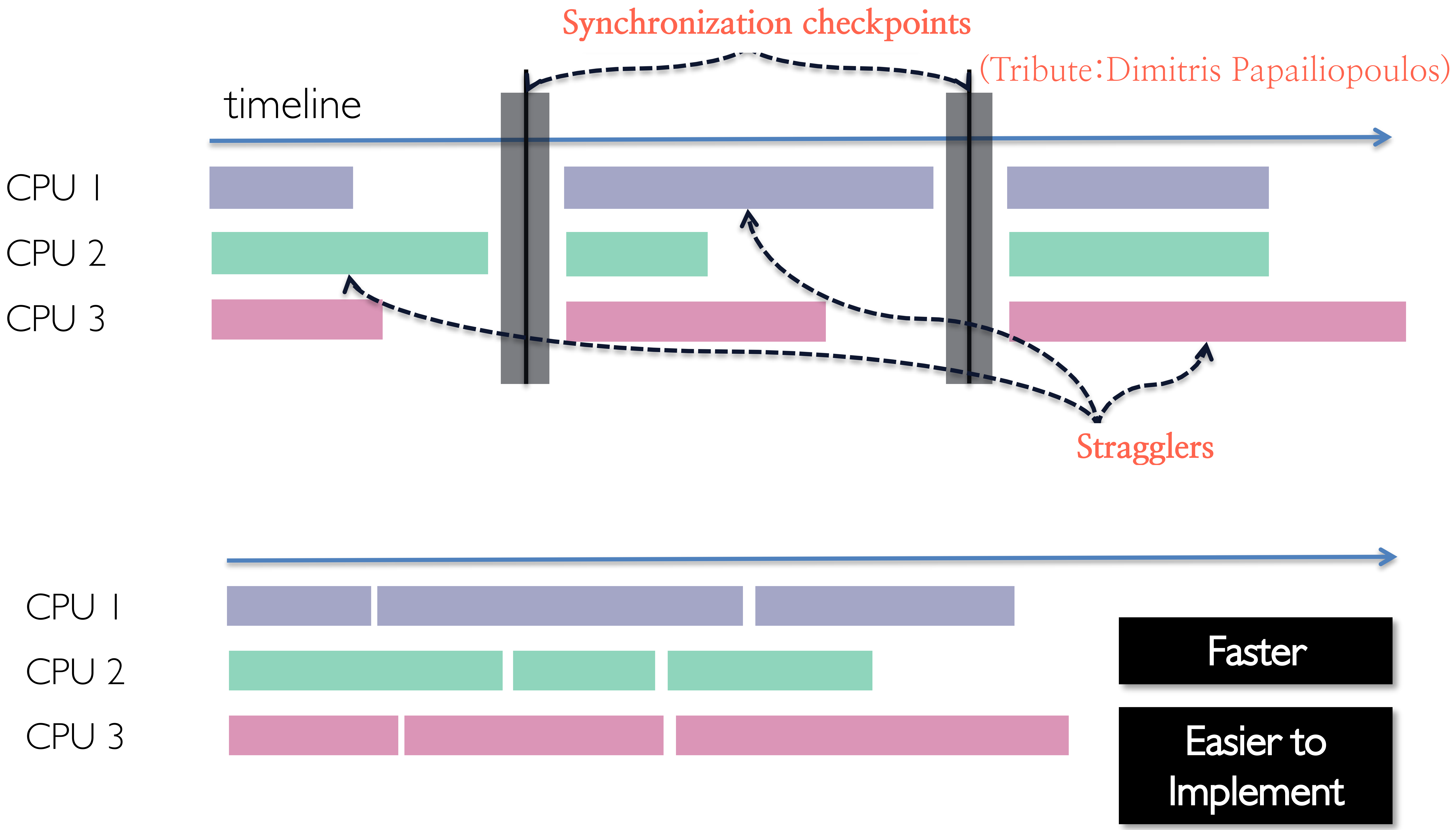

Synchronization checkpoints
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timeline
CPU 1
CPU 2
CPU 3
Stragglers
CPU 1
CPU 2
CPU 3
Faster
Easier to Implement

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- **Alternatives or we have to bear with this situation?**

# Asynchronous distributed computing

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- Main **bottleneck**:
  - Synchronization (locking) amongst processors

# Asynchrony in SGD

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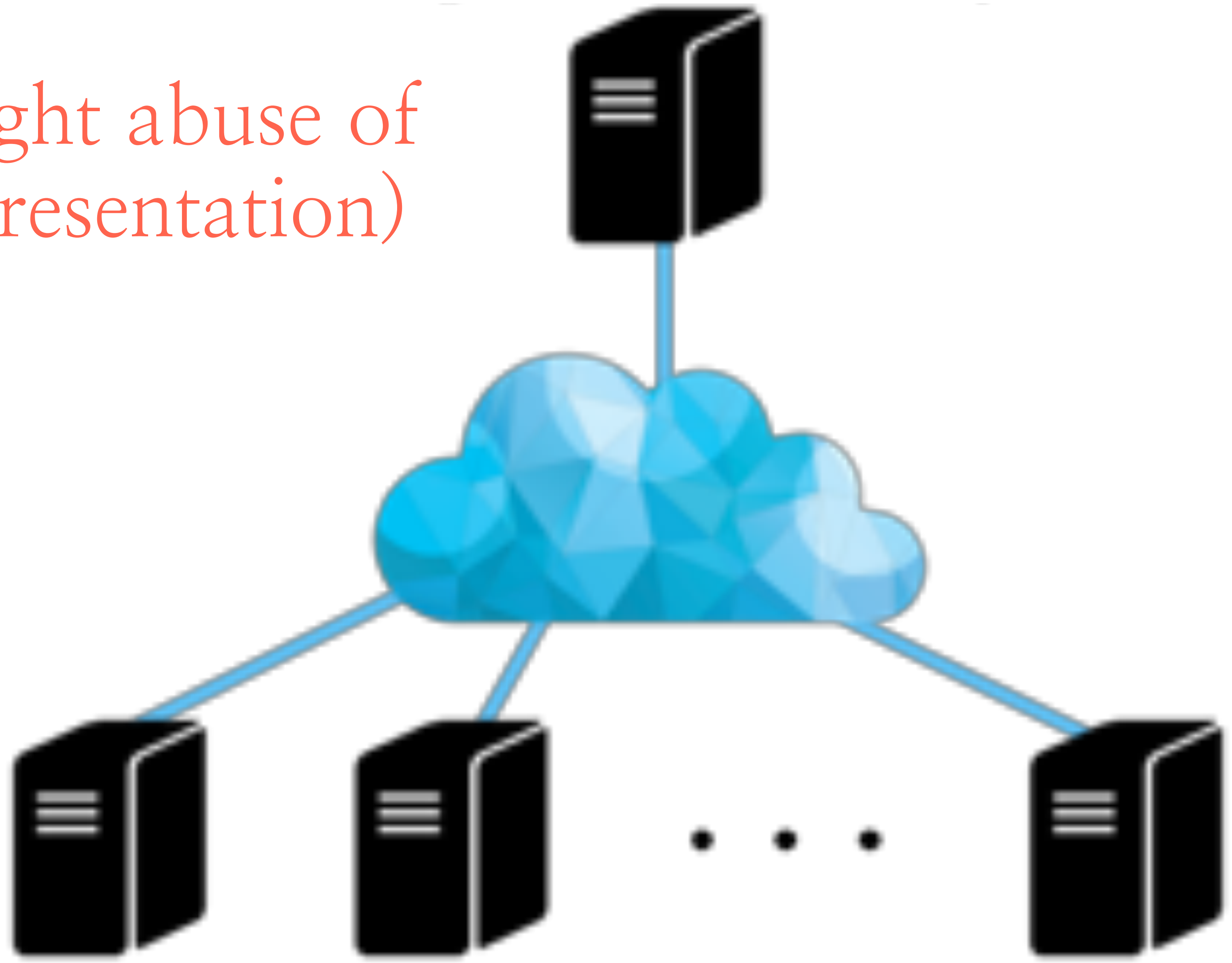
– All threads have access to shared memory

Shared memory

(Slight abuse of representation)



Thread 1 Thread 2 Thread P

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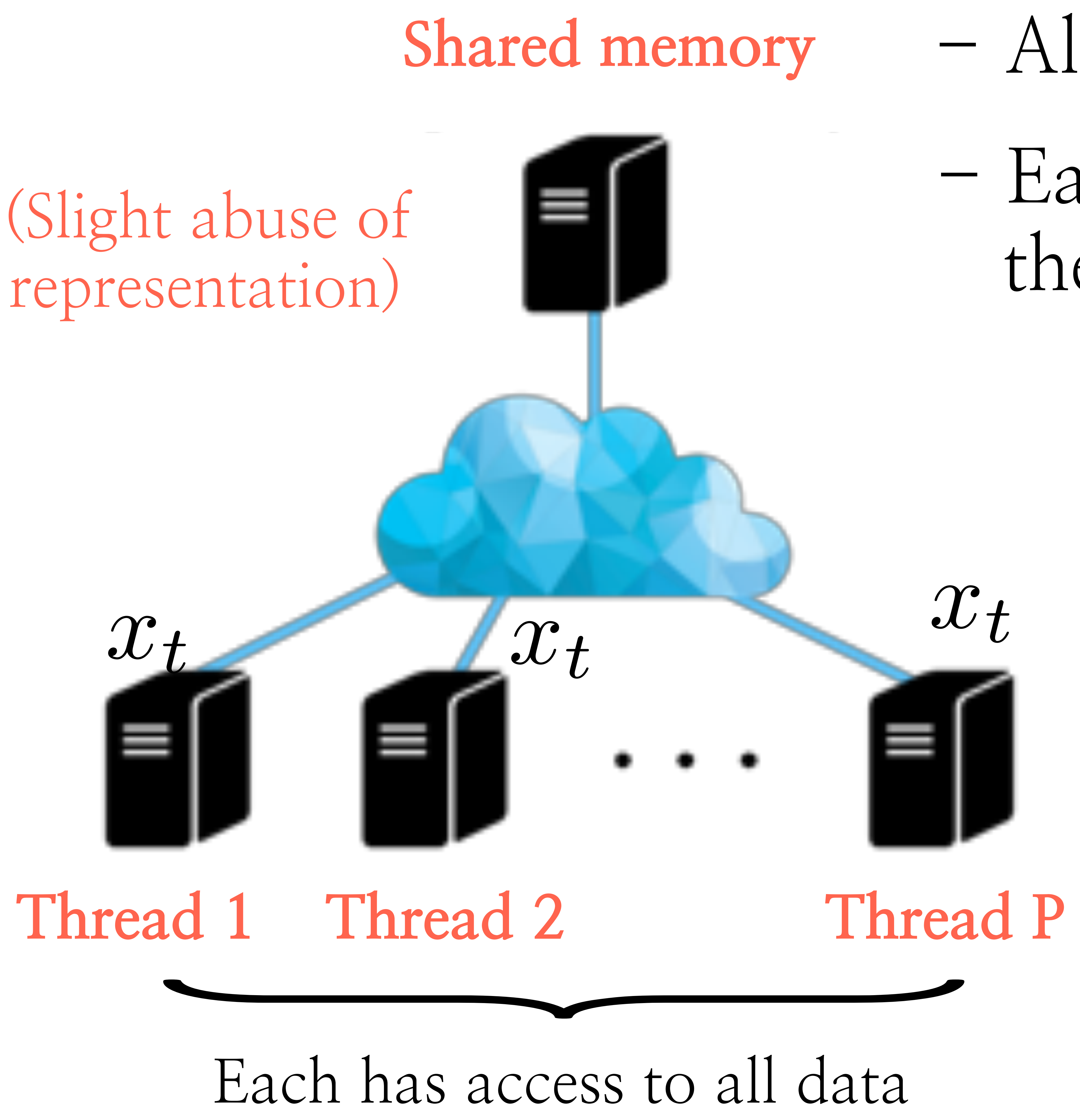


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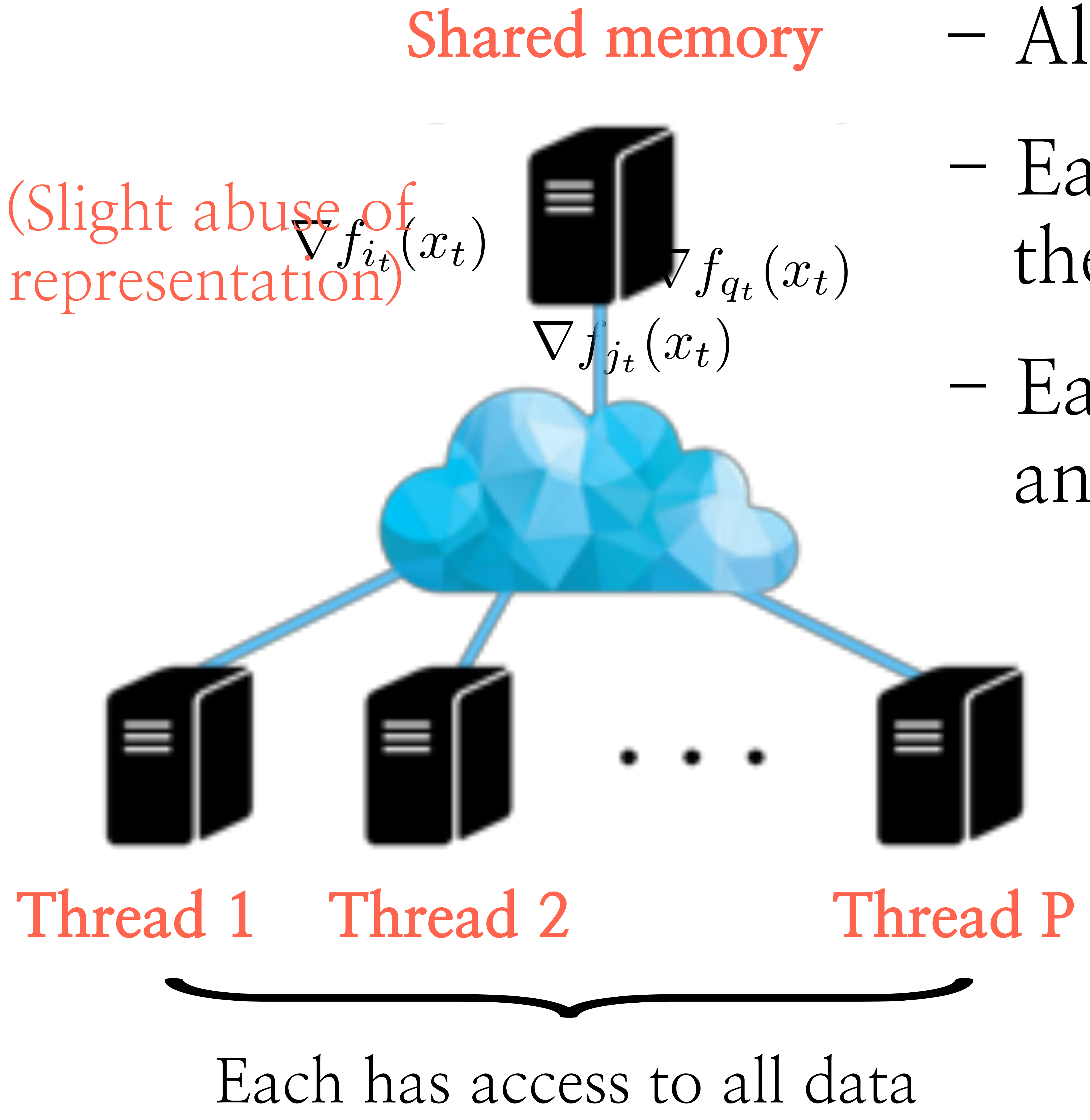


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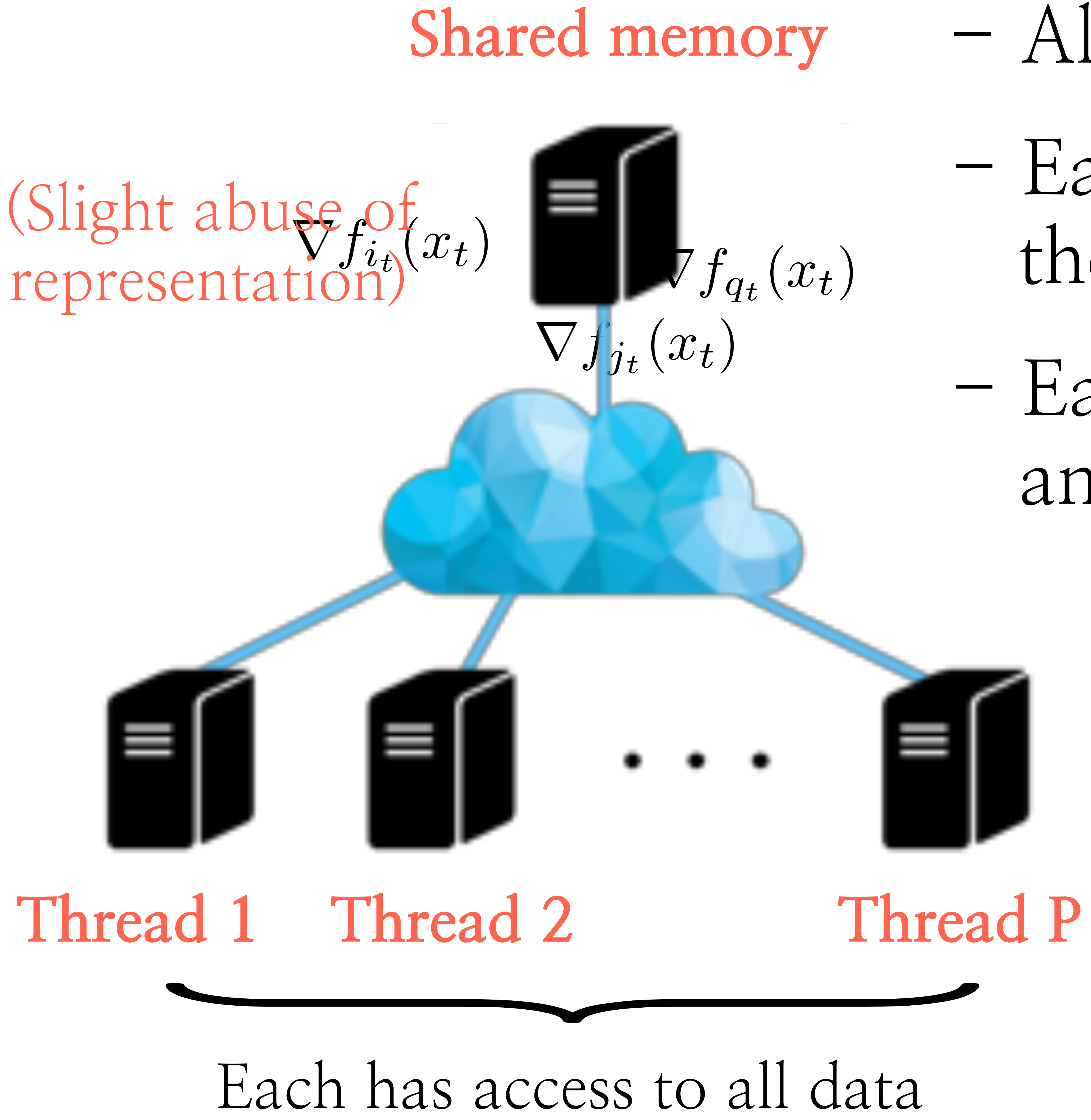


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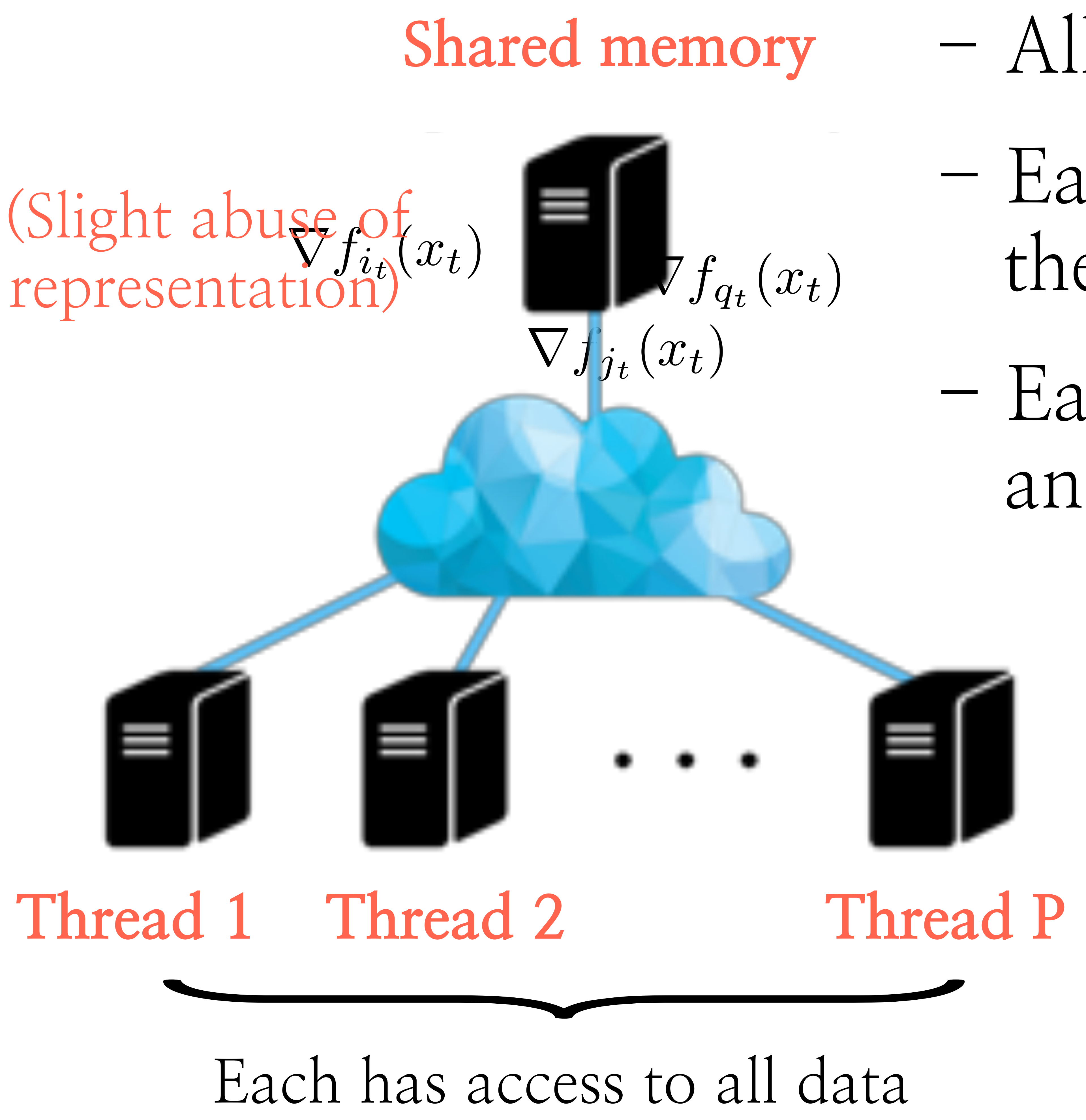


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– Assuming all threads have collected $x_t$

$$x_{t+1} = x_t - \eta\left(\nabla f_{i_t}(x_t) + \nabla f_{j_t}(x_t) + \cdots + \nabla f_{q_t}(x_t)\right)$$

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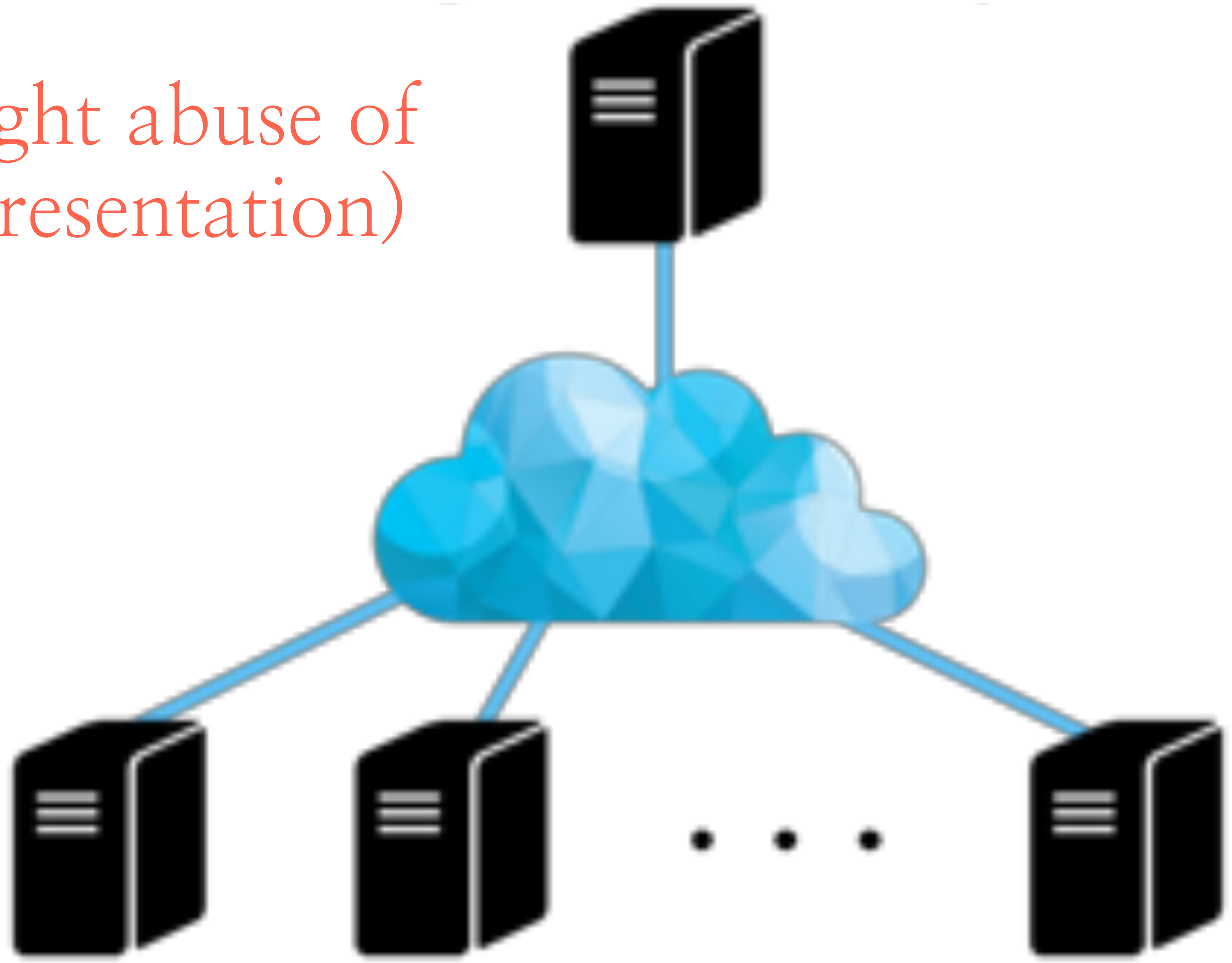

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Google

Microsoft

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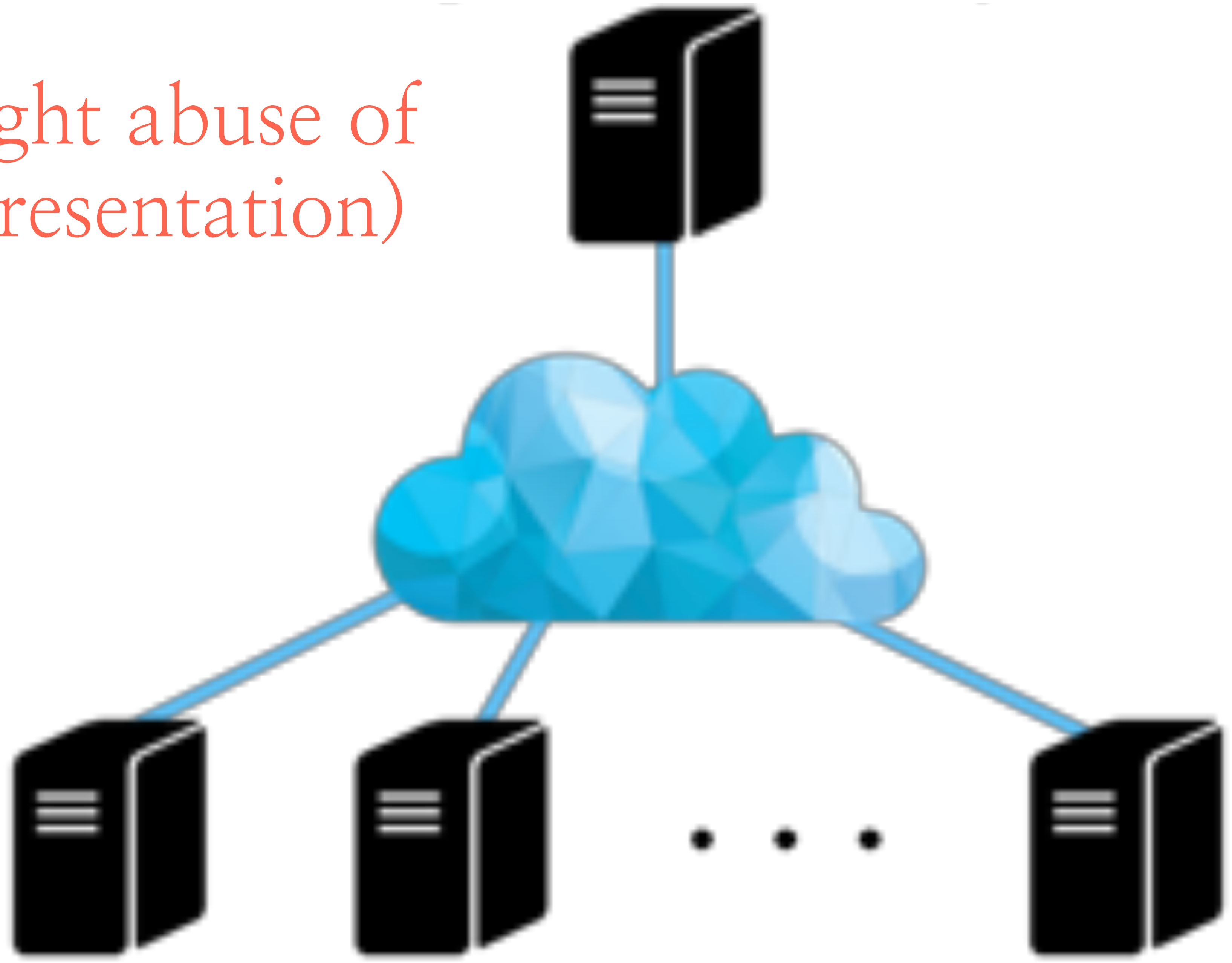

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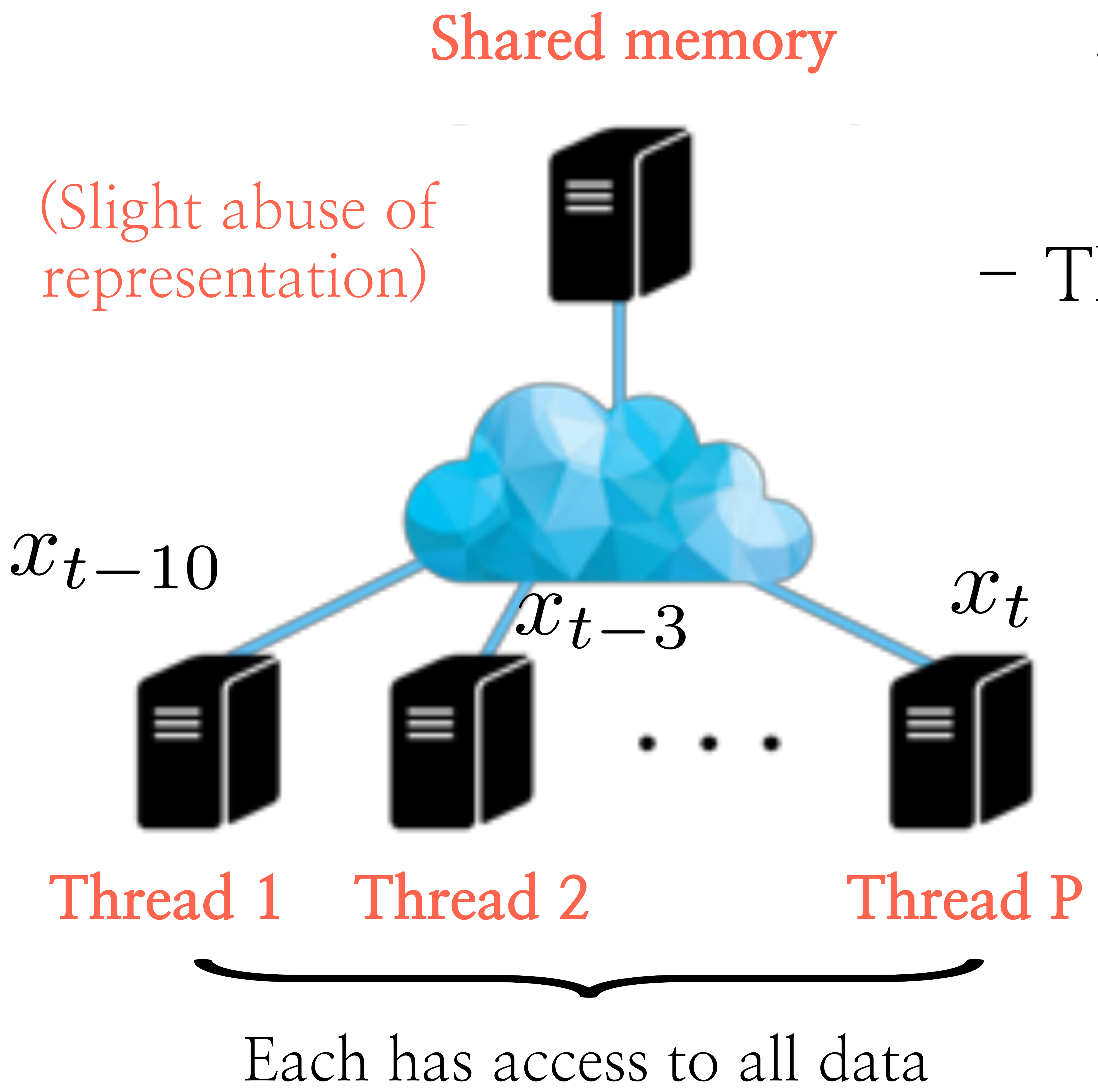


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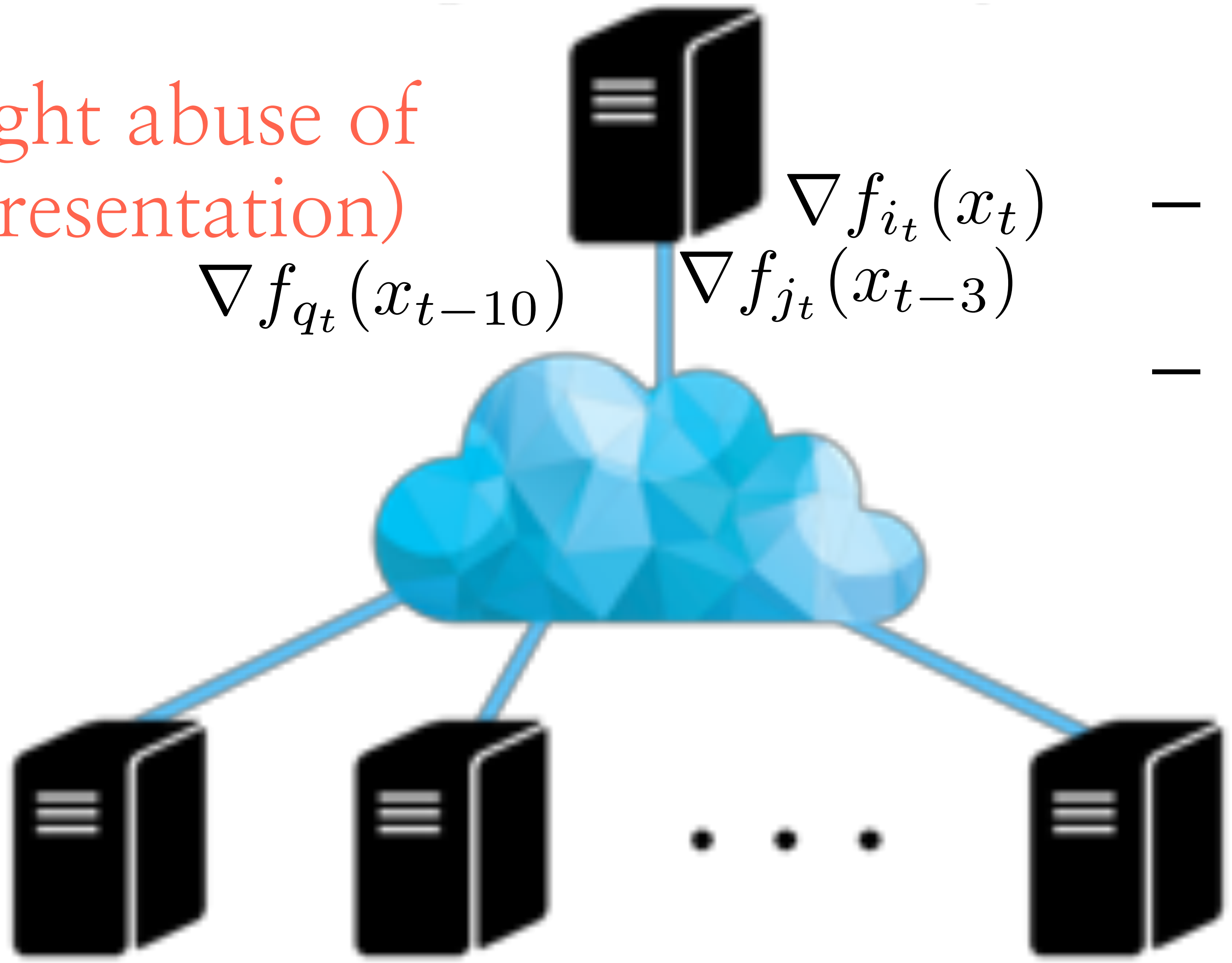


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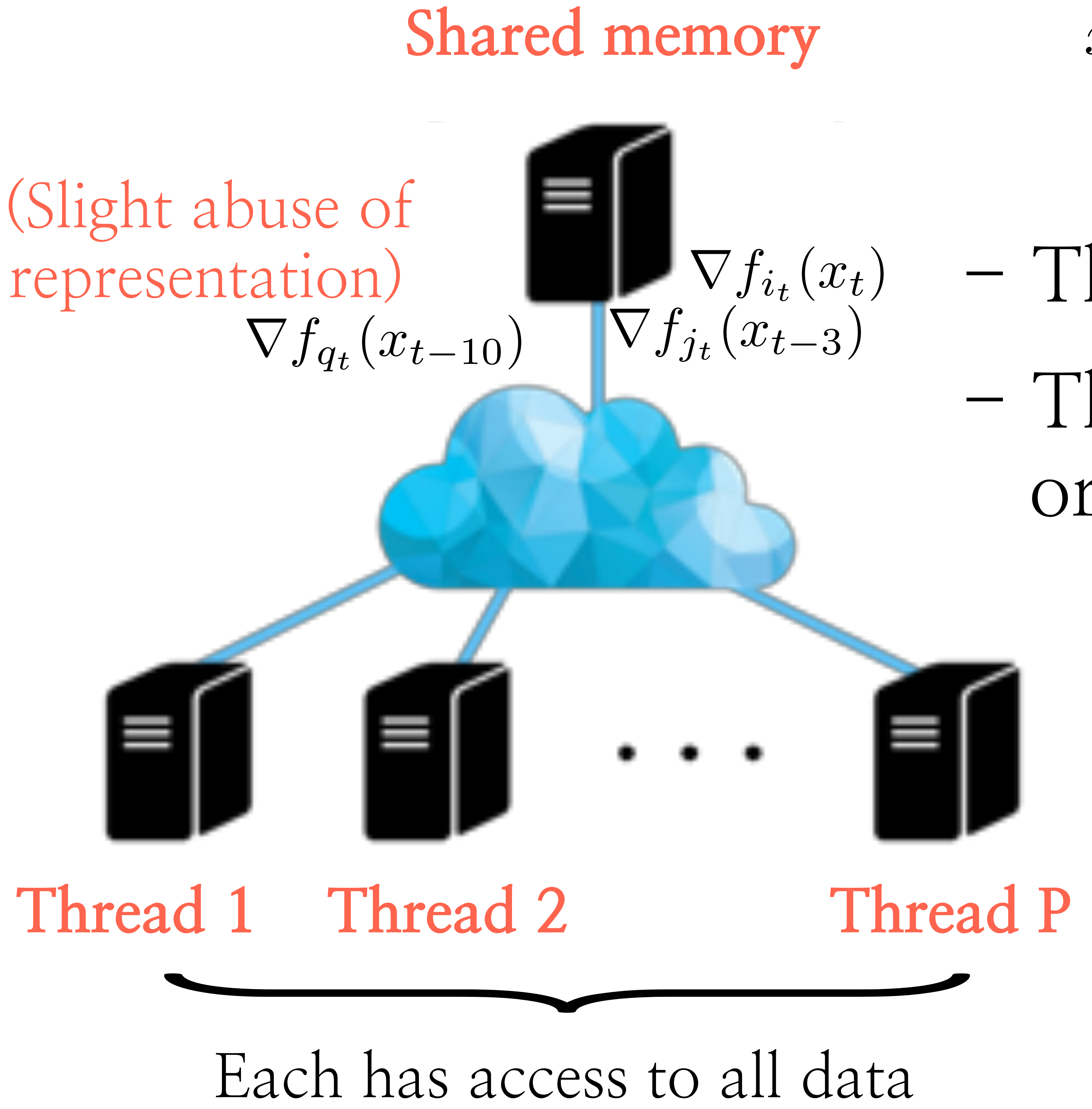


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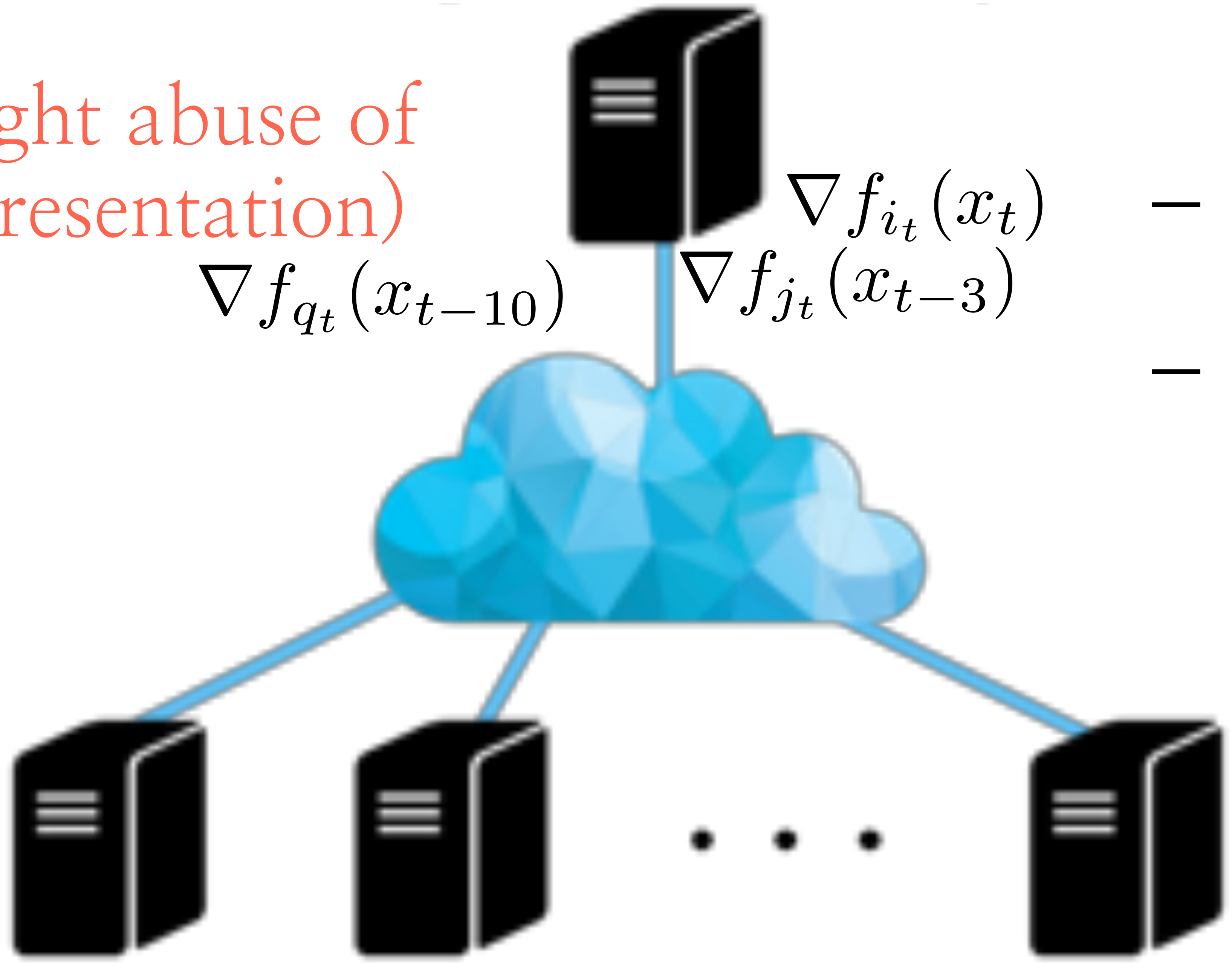


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- And it can get more complex:

  "Threads can read a model state that only stayed in memory for a short time and between other memory writes"

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- Does it work?

# Asynchrony in SGD

(..a bit more involved set up)

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## Large Scale Distributed Deep Networks

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**Matthieu Devin, Quoc V. Le, Mark Z. Mao, Marc'Aurelio Ranzato,**
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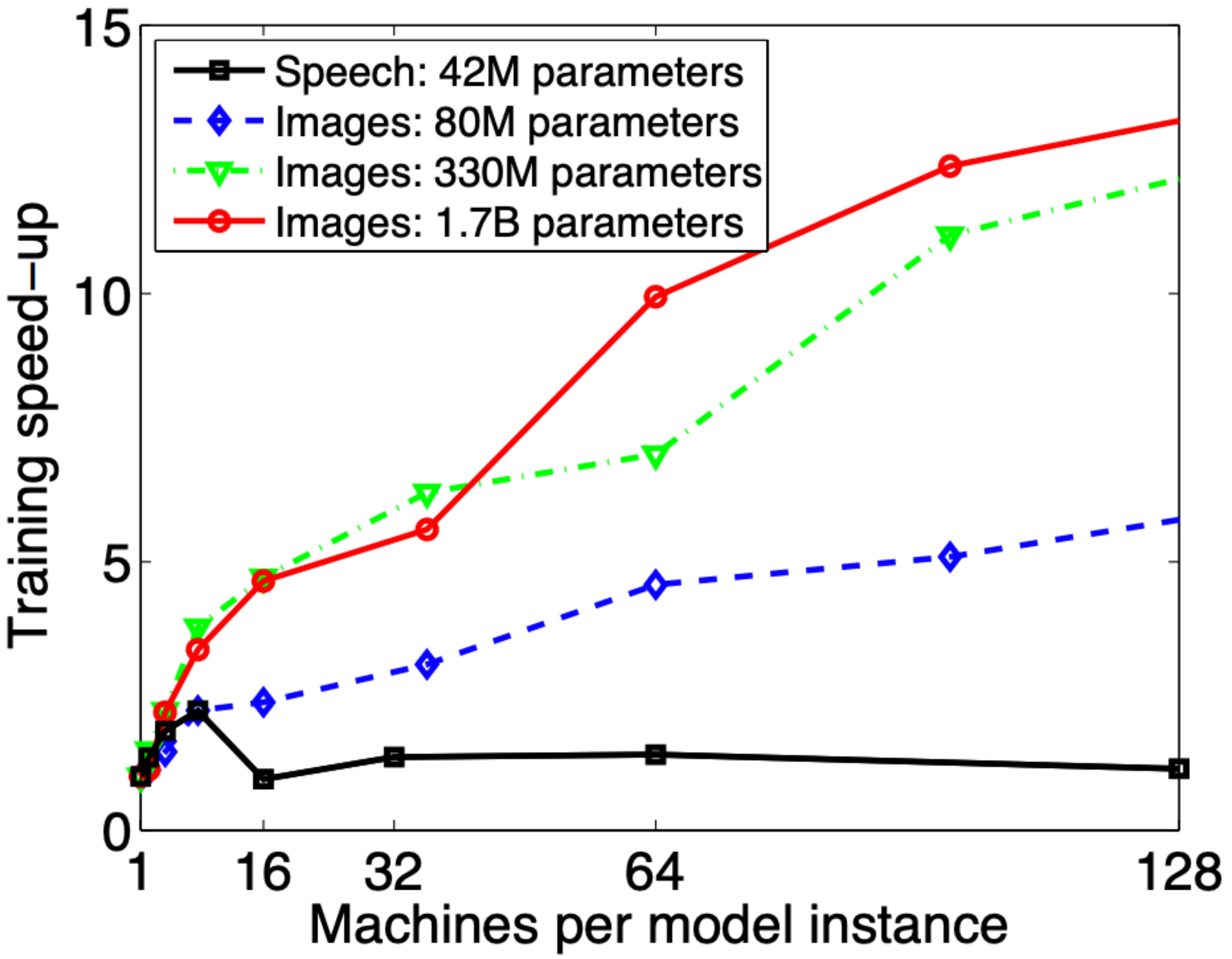


– Comm. bottleneck

– We can increase batch size – but, we deal with worse generali–nation error

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HOGWILD!: "..an update scheme that allows processors accesss shared memory with the possibility of overwriting each other's work

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– Setting: $\min_x f(x) := \sum_{e \in E} f_e(x_e)$

where: $x \in \mathbb{R}^n$ $E$ is a collection of items, say samples

$e \subset [n]$ (each element e is a collection of indices in [n] but also an index from a set of samples E)

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- Slight abuse of notation:

$f_e(\cdot)$: denotes a component of sum of functions, indexed by sample $e$

$x_e$ : corresponds to sub-vector, indexed by an index set $e$ (connected to sample $e$ )

# Asynchrony in SGD

– Key observation: $n$ & $|E|$ are large, while individual $f_e(\cdot)$ act on a small number of components of $x \in \mathbb{R}^n$

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- Observe that, if $z_\alpha$ is very sparse (which happens in reality often), then

$$x^\top z_\alpha = x_\alpha^\top z_\alpha \longleftarrow$$

Main objective depends on a subset of entries

# Asynchrony in SGD

- Some quantities:

  $\Omega$ : maximum number of features involved over all samples

  $\Delta$ : maximum frequency of features that can appear in samples

  $\rho$ : approaches 1 if features are very common across examples

  (..and thus we expect often collisions)

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- Configuration:

  $p$ : number of processors

  Each processor can read model $x$ and contribute an update to $x$

# Asynchrony in SGD

**Algorithm 1** HOGWILD! update for individual processors

1: **loop**
2: Sample $e$ uniformly at random from $E$
3: Read current state $x_e$ and evaluate $G_e(x)$
4: **for** $v \in e$ **do** $x_v \leftarrow x_v - \gamma b_v^T G_e(x)$ (coordinate-wise)
5: **end loop**

- Notation:

$G_e(x) \in \mathbb{R}^n$: gradient with non-zeros indexed by $e$, and scaled such that

$$\mathbb{E}\left[G_e(x_e)\right] = \nabla f(x)$$

Observe that $[G_e(x_e)]_{e^c} = 0$

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– In words:

1. Each processor samples $e$ uniformly at random
2. Each processor computes the gradient $f_e$ at $x_e$
3. Each processor applies update on each coordinate in $e$

# Asynchrony in SGD

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Whiteboard

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No Demo (no resources)

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  - This further indicates that **asynchrony introduces barely any error in the computations**
  - The authors show (theoretically and experimentally) a near-linear speedup, with the number of processors used
  - In practice, lock-free SGD exceeds even theoretical guarantees

# Alternatives to avoid asynchrony

(in other words, how we can decrease communication burden?)

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**QSGD: Communication-Efficient SGD via Gradient Quantization and Encoding**


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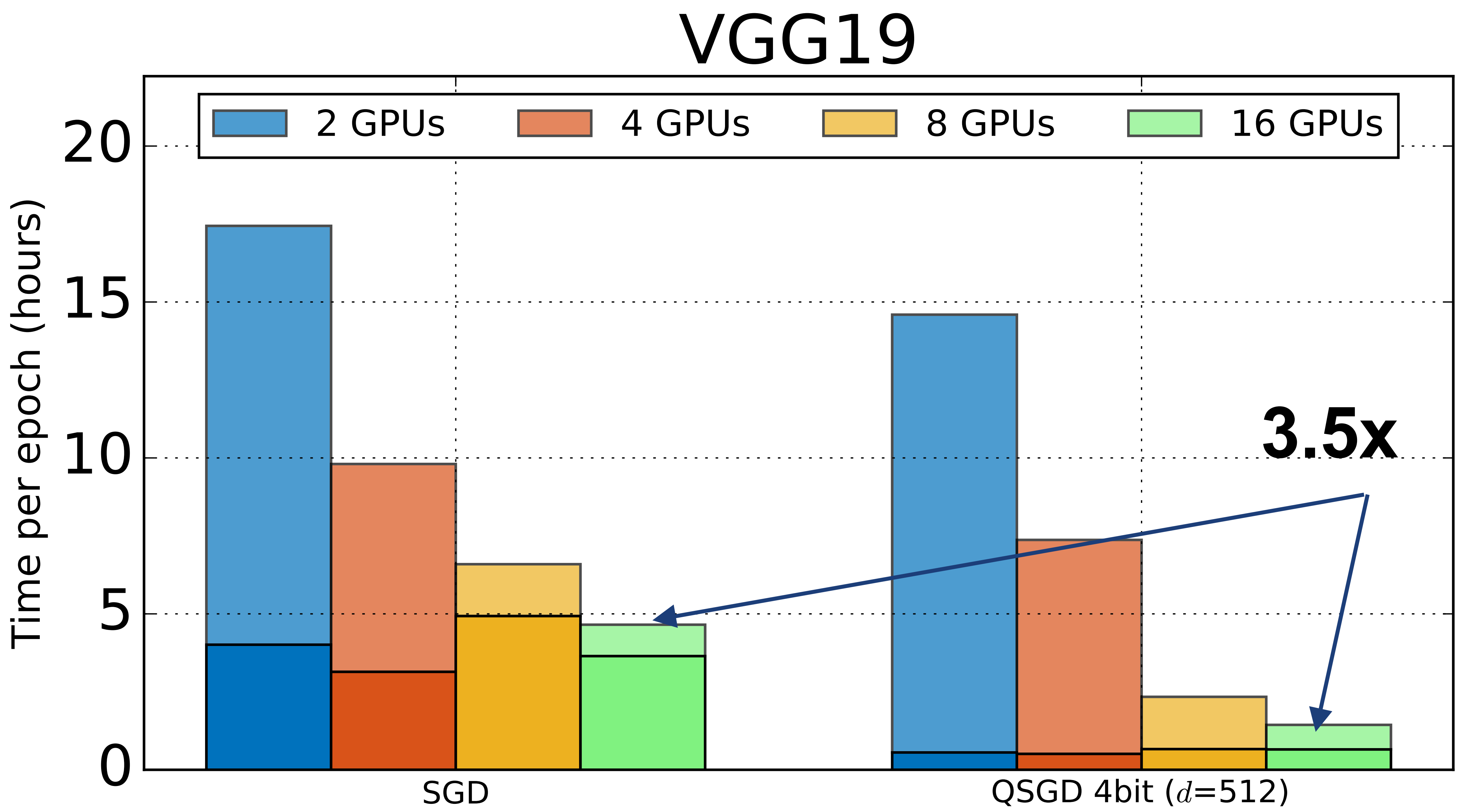

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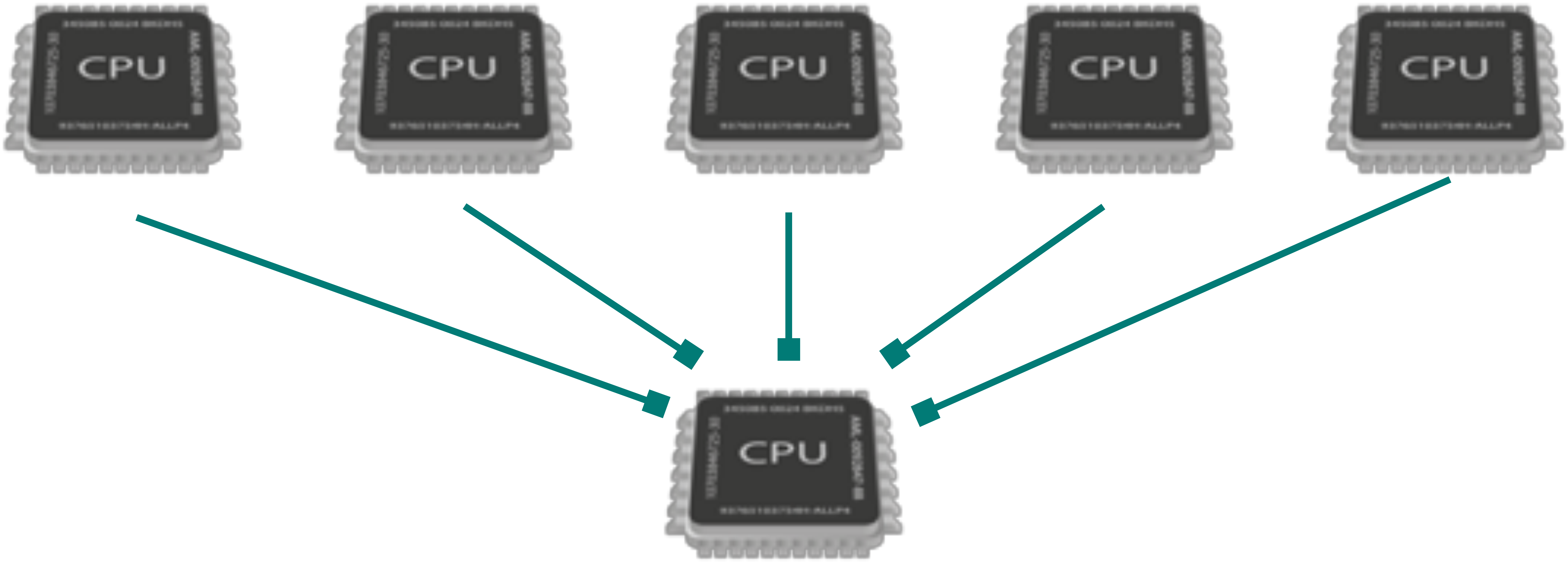


- Right learner can slow down the performance of synchronized SGD

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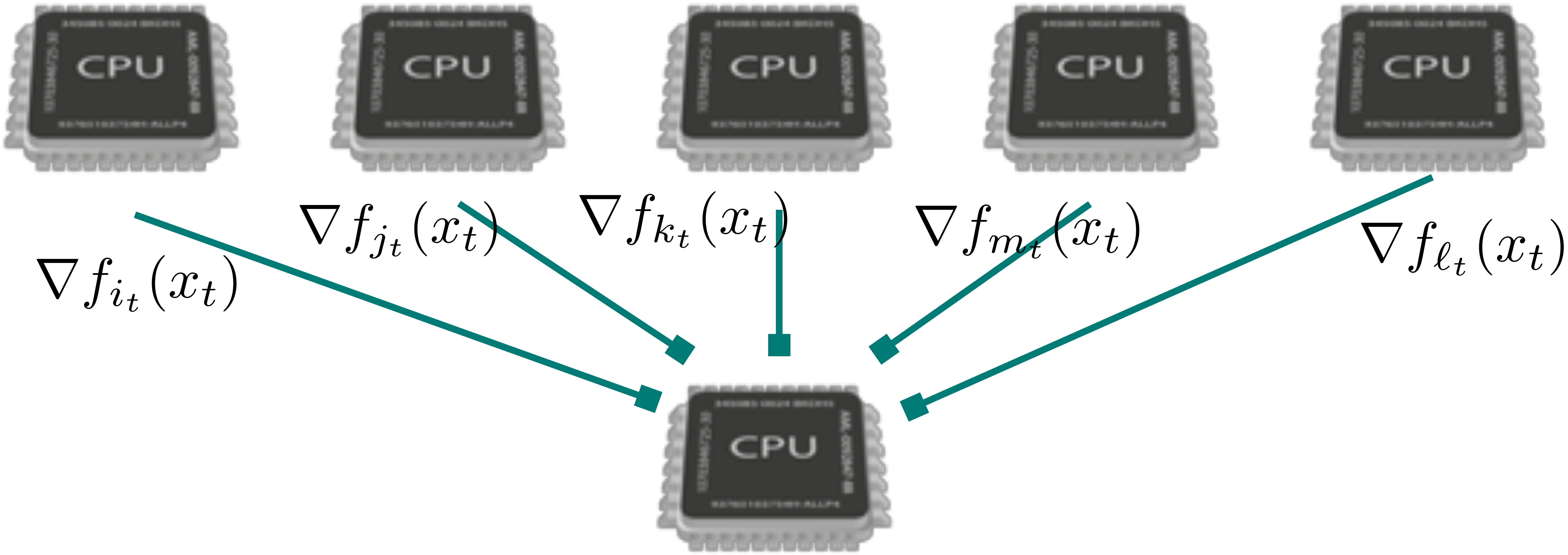


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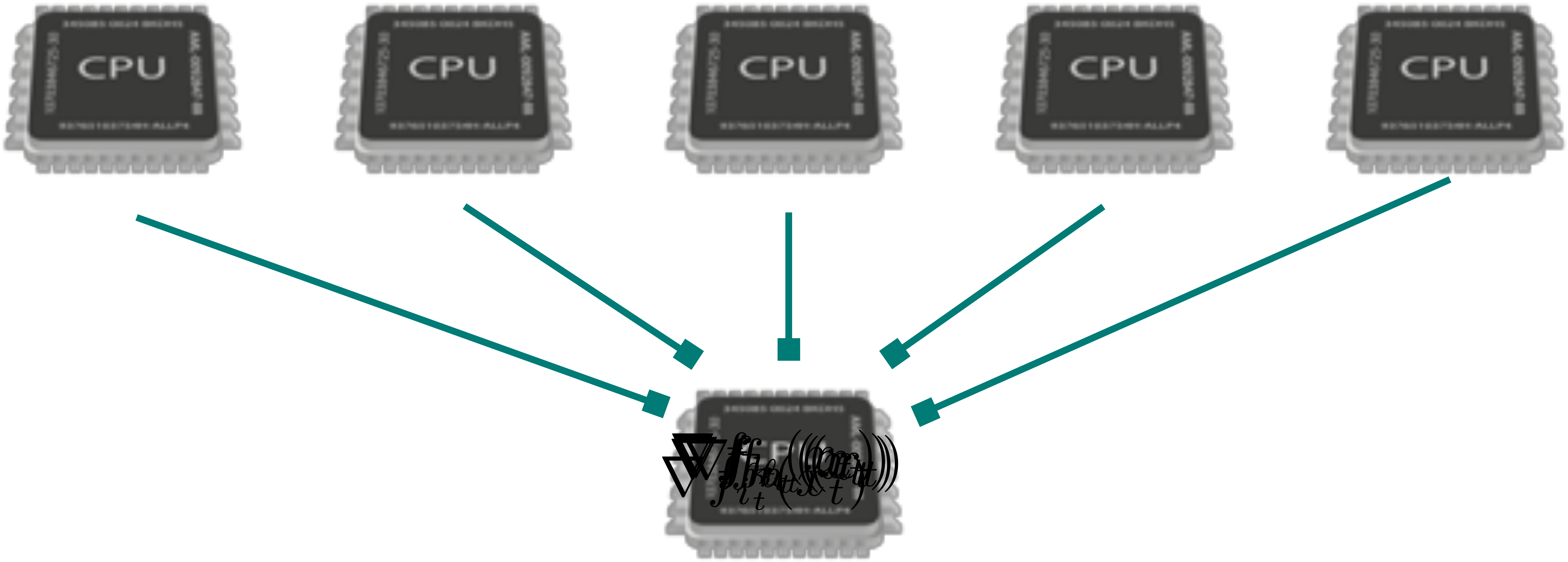


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## Revisiting Distributed Synchronous SGD

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### Abstract

Distributed training of deep learning models on large-scale training data is typically conducted with *asynchronous* stochastic optimization to maximize the rate of updates, at the cost of additional noise introduced from asynchrony. In contrast, the *synchronous* approach is often thought to be impractical due to idle time wasted on waiting for straggling workers. We revisit these conventional beliefs in this paper, and examine the weaknesses of both approaches. We demonstrate that a third approach, synchronous optimization with backup workers, can avoid asynchronous noise while mitigating for the worst stragglers. Our approach is empirically validated and shown to converge *faster* and to *better* test accuracies.

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- **Large batch training:** give more ``work`` to workers by increasing the batch size. However it needs careful parameter tuning to make it work

- **Variants of HOGWILD! that minimize communication conflicts:** some computation is performed to distribute examples to different cores so that examples do not ``conflict``.

# Conclusion

- Distributed computing is at the heart of developments in modern ML
- There are different ways to exploit distributed computing: **hyper parameter optimization, coordinate descent, mini-batch synchronous** SGD, **asynchronous** SGD
- Which configuration to use depends on the problem and the resources at hand
- These topics are highly attractive (research-wise): they define the notion of systems + machine learning (look for SysML conference)